§6.3 p359 Problem 9. Health Care Plan Options. A new state employee is offered a choice of ten basic health plans, three dental plans, and two vision care plans. How many different health-care plans are there to choose from if one plan is selected from each category?

Solution. To choose a complete health-care plan, the employee must choose a basic plan, choose a dental plan, and choose a vision-care plan. Symbolically, we can think of filling in slots as shown below.

10 choices 3 choices 2 choices
Basic Dental Vision

Since there are 10 ways to make the first choice, 3 ways to make the second, and 2 ways to make the third, there are

\[ 10 \times 3 \times 2 = 60 \]

ways to make all three choices, hence 60 possible health-care plans.

§6.3 p359 Problem 13. Computer Dating. A computer dating service uses the results of its compatibility survey for arranging dates. The survey consists of 50 questions, each having five possible answers. How many different responses are possible if every question is answered?

Solution. We are trying to count how many ways one could complete the entire survey, being sure to answer each of the 50 questions. Since there are 5 ways to answer the first question, 5 ways to answer the second question, and so forth, with 50 questions in all, the so-called Multiplication Principle tells us that there are

\[ 5 \times 5 \times \cdots \times 5 \]

50 factors

ways to complete the entire survey. In other words, there are \(5^{50}\) ways (very many!) to complete the survey.

§6.3 p359 Problem 17. ATM Cards. To gain access to his account, a customer using an ATM machine must enter a four-digit code. If repetition of the same four digits is not allowed (for example, 5555 is not allowed) how many possible codes are there?

Solution. This time it will be easier to count all possible four-digit codes, count the ones that cannot be used, and then subtract to get a count of the ones that can be used.
To count all possible codes, note that creating a code involves choosing a digit for the first position, choosing a digit for the second position, and so forth. Since there are 10 choices (0 through 9) for each position, there are
\[ 10 \times 10 \times 10 \times 10 = 10^4 = 10,000 \]
possible (unrestricted) codes.

How many of these codes are excluded? The excluded ones are 0000, 1111, 2222, and so forth, up through 9999. Thus exactly 10 codes are excluded.

Subtracting, we see that the number of allowable codes is
\[ 10^4 - 10 = 10,000 - 10 = 9,990. \]

§6.3 p359 Problem 21. Exams. An exam consists of ten true-or-false questions. Assuming that every question is answered, in how many different ways can a student complete the exam? In how many ways may the exam be completed if a penalty is imposed for each incorrect answer, so that a student may leave some questions unanswered?

Solution. This is exactly like the dating survey question in Problem 13.

First consider the case where the student answers each of the 10 questions T or F. She must choose how to answer the first question, then choose how to answer the second question, and so forth, on through the tenth and last question. Since she has 2 choices for each question, the situation looks like

\[
\begin{array}{cccc}
2 \text{ choices} & 2 \text{ choices} & \cdots & 2 \text{ choices} \\
\text{Question 1} & \text{Question 2} & \cdots & \text{Question 10}
\end{array}
\]

Thus she has
\[ 2 \times 2 \times \cdots \times 2 \]
\[ 10 \text{ factors} \]
ways to complete the exam. In other words, there are \(2^{10} = 1024\) ways for her to complete the exam.

Now consider the case where she has the option of leaving some of the questions blank. What has changed is that for each of the 10 questions, she now has 3 choices instead of 2. Following the same reasoning as before, we see that there are now \(3^{10} = 59,049\) ways to complete the exam, many more possibilities.
§6.3 p359 Problem 23. Lotteries. In a state lottery there are 15 finalists eligible for the Big Money draw. In how many ways can the first, second, and third prizes be awarded if no ticket holder may win more than one prize?

Solution. To choose a particular set of winners, the lottery folk could choose a third-place winner from the original 15 finalists, then choose a second-place winner from the remaining 14 finalists, and then choose a first-place winner from the remaining 13. By the multiplication principle, the total number of ways to choose all three winners (or award all three prizes) is  \( P(15, 3) = 15 \times 14 \times 13 = 2730 \).

§6.3 p359 Problem 25. Slot Machines. A “lucky” dollar” is one of the nine symbols printed on each reel of a slot machine with three reels. A player receives one of various payouts whenever one or more lucky dollars appear in the window of the machine. Find the number of winning window displays for which the machine gives a payoff.

Solution. Once again, it will be easier to count the ones we don’t want. In other words, count all the possible window displays, count the window displays that do not show a lucky dollar, and then subtract to find out how many window displays do display a lucky dollar. (In other words, we’re counting the universal set, counting the set of nonwinning displays, and subtracting to find the count for the complementary set of winning displays.) We will soon see that the two separate counts we do before subtracting involve the same ideas as are used for the TF test and the dating survey.

So how many displays are there in all? There are three windows, each one of which could be filled with any one of the 9 symbols on the accompanying reel. In other words, we have

\[
\begin{align*}
\text{Reel 1} & \quad 9 \text{ choices} \\
\text{Reel 2} & \quad 9 \text{ choices} \\
\text{Reel 3} & \quad 9 \text{ choices}
\end{align*}
\]

Thus there are

\[ 9 \times 9 \times 9 = 9^3 = 729 \]

possible displays in all.

How many of the displays do not show a lucky dollar? This time we still have three windows, but now each one of the windows could be filled with any one of the 8 symbols that are not the lucky dollar. Exactly the same reasoning as before tells us that there are

\[ 8 \times 8 \times 8 = 8^3 = 512 \]

displays that do not win.

Now subtract to find that the number of possible winning combinations is

\[ 729 - 512 = 217 \].