§7.5 p 358 Problem 4. Two fair dice are rolled. Find the probability the the sum is 8, given that the sum is greater than 7.

Solution. Use the sample space described in Example 2 on pp 336-337, where there are 36 equally likely outcomes (and use the table on page 337 as an aid for counting outcomes). Of these 36, there are 15 in which the sum is greater than 7. Of these, there are 5 in which the sum is 8. Therefore

\[ P(\text{sum } = 8 \text{ and sum } > 7) = P(\text{sum } = 8) = \frac{5}{36} \]
\[ P(\text{sum } > 7) = \frac{15}{36} \]
\[ P(\text{sum } = 8 \mid \text{sum } > 7) = \frac{\frac{5}{36}}{\frac{15}{36}} = \frac{5}{15} = \frac{1}{3} \]

§7.5 p 358 Problem 5. Two fair dice are rolled. Find the probability that the sum is 6, given that the roll was a double (two identical numbers).

Solution. Use the sample space described in Example 2 on pp 336-337, where there are 36 equally likely outcomes (and use the table on p337 for counting). Let \( D \) denote the event that the roll is a double. Let \( S_6 \) denote the event that the sum is 6. Then

\[ P(S_6 \mid D) = \frac{P(S_6 \cap D)}{P(D)} = \frac{\frac{1}{36}}{\frac{6}{36}} = \frac{1}{6} \]

§7.5 p 358 Problem 6. Two fair dice are rolled. Find the probability that the roll was a double, given that the sum was 9.

Solution. Use the sample space described in Example 2 on pp 336-337, where there are 36 equally likely outcomes. Let \( D \) denote the event that the roll is a double. Let \( S_9 \) denote the event that the sum is 9. Then

\[ P(D \mid S_9) = \frac{P(D \cap S_9)}{P(S_9)} = 0 \]

since there are no outcomes in which the roll is a double and the sum is 9.

§7.5 p 358 Problem 7. Two cards are drawn (in succession but) without replacement from an ordinary 52-card deck of playing cards. Find the probability that the second is a heart, given that the first is a heart. (Assume the cards are drawn at random.)

Solution. We are given that the first card drawn is one of the 13 hearts. At the second draw,
there are only 51 cards left, of which only 12 are hearts (since the heart that was drawn took one of the 13 hearts away). So the probability of getting a heart on the second draw, given a heart on the first draw, will be \( \frac{12}{51} \) or \( \frac{4}{17} \).

\section*{7.5 p 358 Problem 8.} Two cards are drawn without replacement from an ordinary 52-card deck of playing cards. Find the probability that the second is black, given that the first is a spade.

\textbf{Solution.} We are given that the first card drawn is a spade, which is a black card. At the second draw there are only 51 cards left, 25 of which are black (since the spade that was already drawn took one of the 26 black cards away). So the probability of getting a spade on this second draw (given our assumption that the first card is a spade) will be \( \frac{25}{51} \).

\section*{7.5 p 358 Problem 23.} Let \( A \) and \( B \) be independent events with \( P(A) = \frac{1}{4} \) and \( P(B) = \frac{1}{5} \). Find \( P(A \cap B) \) and \( P(A \cup B) \).

\textbf{Solution.} Since \( A \) and \( B \) are independent, the product rule for independent events gives us

\[ P(A \cap B) = P(A)P(B) = \left( \frac{1}{4} \right) \left( \frac{1}{5} \right) = \frac{1}{20} = 0.05. \]

Then, the union rule gives us

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{1}{4} + \frac{1}{5} - \frac{1}{20} = \frac{8}{20} = \frac{2}{5} = 0.4. \]

\section*{7.5 p 358 Problem 24.} Suppose \( A \) and \( B \) are events such that \( P(A) = 0.5 \) and \( P(A \cup B) = 0.8 \).

(a) Find \( P(B) \) when \( A \) and \( B \) are mutually exclusive.

(b) Find \( P(B) \) when \( A \) and \( B \) are independent.

\textbf{Solution.} (a) For mutually exclusive events \( A \) and \( B \), we know that \( P(A \cup B) = P(A) + P(B) \). So in this case we have

\[ P(B) = P(A \cup B) - P(A) = 0.8 - 0.5 = 0.3. \]

(b) We always have that

\[ P(A \cup B) = P(A) + P(B) - P(A \cap B). \]

For independent events \( A \) and \( B \) we also have

\[ P(A \cap B) = P(A)P(B). \]
So in this case we have

\[ P(A \cup B) = P(A) + P(B) - P(A)P(B). \]

Substituting the given values for \( P(A) \) and \( P(A \cup B) \), we find

\[ 0.8 = 0.5 + P(B) - (0.5)P(B) \]

\[ (0.5)P(B) = 0.3 \]

\[ P(B) = \frac{0.3}{0.5} = \frac{3}{5} = 0.6 \]

§7.5 p 358 Problem 31. Backup Computers

Corporations where a computer is essential to day-to-day operations, such as banks, often have a second backup computer in case the main computer fails. Suppose there is a .003 chance that the main computer will fail in a given time period, and a .005 chance that the backup computer will fail while the main computer is being repaired. Assume these failures represent independent events, and find the fraction of the time that the corporation can assume it will have computer service. How realistic is our assumption of independence?

**Solution.** We want to find the fraction of the time that at least one of the two computers will be functional. In other words, we are asking: in any given time period, what is the probability that at least one of the two computers will be functional? The complementary event is that both computers are failing. Since the two failures are considered to be independent events, the probability of both failing is the product of the individual failure probabilities, which is \((0.003)(0.005)\). Therefore the probability we want is given by

\[ 1 - (0.003)(0.005) = 1 - .000015 = 0.999985. \]

I’d say the independence assumption is somewhat realistic, but not entirely so. The machines are different machines, each likely to fail now and then but not likely to fail simultaneously. On the other hand, a power blackout such as affected parts of the East coast last summer might well affect both computers. You may have additional thoughts.
§7.5 p 358 Problem 61. Rain Forecasts  In a letter to the journal *Nature* (Vol. 382, Aug 29, 1996, p3), Robert A. J. Matthews gives the following table of outcomes of forecast and weather over 1000 one-hour walks, based on the United Kingdom’s Meteorological office’s 83% accuracy in 24-hour forecasts.

<table>
<thead>
<tr>
<th>Forecast of Rain</th>
<th>Rain</th>
<th>No Rain</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>66</td>
<td>156</td>
<td>222</td>
</tr>
<tr>
<td>Forecast of No Rain</td>
<td>14</td>
<td>764</td>
<td>778</td>
</tr>
<tr>
<td>Sum</td>
<td>80</td>
<td>920</td>
<td>1000</td>
</tr>
</tbody>
</table>

(a) Verify that the probability that the forecast called for rain, given that there was rain, is indeed 83%. Also verify that the probability that the forecast called for no rain, given that there was no rain, is also 83%.

(b) Calculate the probability that there was rain, given that the forecast called for rain.

(c) Calculate the probability that there was no rain, given that the forecast called for no rain.

(d) Observe that your answer to part (c) is higher than 83%, and that your answer to part (b) is much lower. Discuss which figure best describes the accuracy of the weather forecast in recommending whether or not you should carry an umbrella.

**Solution.** Throughout this problem, let $F$ denote the event that rain was forecast, and let $R$ denote the event that it rained.

(a)  

$$P(F \mid R) = \frac{P(F \cap R)}{P(R)} = \frac{n(F \cap R)}{n(R)} = \frac{66}{80} = .825.$$  

I guess you could call that 83%.

$$P(F' \mid R') = \frac{P(F' \cap R')}{P(R')} = \frac{n(F' \cap R')}{n(R')} = \frac{764}{920} \approx .830435.$$  

You could call that 83%.

(b)  

$$P(R \mid F) = \frac{P(R \cap F)}{P(F)} = \frac{n(R \cap F)}{n(F)} = \frac{66}{222} \approx 0.297297.$$  

So it’s about 30%.

(c)  

$$P(R' \mid F') = \frac{P(R' \cap F')}{P(F')} = \frac{n(R' \cap F')}{n(F')} = \frac{764}{778} = .982005.$$  

So this is over 98%.
§7.5 p 358 Problem 65. Real Estate A real estate agent trying to sell you an attractive beachfront house claims that it will not collapse unless it is subjected simultaneously to extremely high winds and extremely high waves. According to weather service records, there is a .001 probability of extremely high winds, and the same for extremely high waves. The real estate agent claims that, therefore, the probability of both occurring is (.001)(.001) = .000001. What is wrong with the agent’s reasoning?

Solution. Extremely high winds and extremely high waves often occur at the same time, namely when there’s a heavy storm. So the event that the house will be subjected to extremely high winds and the event that it will be subjected to extremely high waves are definitely not independent. So multiplying the two probabilities, as the agent did, gives inaccurate results. The true probability of both events occurring would be considerably higher.

See the next page for Problem 69.

§7.5 p 358 Problem 72. Studying A teacher has found that the probability that a student studies for a test is .6, the probability that a student gets a good grade on a test is .7, and the probability that both occur is .52. Are these events independent?

Solution. For the two events to be independent, the probability that the student studies and gets a good grade should be the product (0.6)(0.7). In other words, it should be the probability she studies times the probability she gets a good grade. But (0.6)(0.7) = 0.42 ≠ 0.52, so the events are not independent. Surprise, surprise.....
§7.5 p 358 Problem 69. Driver's License Test The Motor Vehicle Department has found that the probability of a person passing the test for a driver’s license on the first try is .75. The probability that an individual who fails on the first test will pass on the second try is .80, and the probability that an individual who fails the first and second tests will pass the third time is .70. Find the probabilities that an individual will do the following.

(a) Fail both the first and second tests

(b) Fail three times in a row

(c) Require at least two tries

Solution. It may be helpful to use a tree diagram here.

(a) Follow the path that shows a failure followed by a failure. The probability is $(0.25)(0.20) = 0.05$.

(b) Follow the path that shows a failure followed by a failure followed by another failure. The probability is $(0.25)(0.20)(0.30) = 0.015$.

(c) Requiring at least two tries is equivalent to failing the first time, so the probability is 0.25.