Problem 11, §5.5, p395. Find \( \int \sqrt{x - 1} \, dx \).

Solution. This just needs a straightforward \( u \)-substitution.

Let \( u = x - 1 \), which gives \( du = dx \). Then

\[
\int \sqrt{x - 1} \, dx = \int u^{1/2} \, du = \frac{2}{3} u^{3/2} + C = \frac{2}{3} (x - 1)^{3/2} + C
\]

This result can also be written as

\[
\int \sqrt{x - 1} \, dx = \frac{2}{3} (x - 1) \sqrt{x - 1} + C
\]

Problem 15, §5.5, p395. Find \( \int \frac{1 + 4x}{\sqrt{1 + x + 2x^2}} \, dx \).

Solution. The key here is to notice that the numerator is exactly the derivative of the expression inside the square root sign. This suggests putting

\[
u = 1 + x + 2x^2
\]

which gives \( du = (1 + 4x) \, dx \). So

\[
\int \frac{1 + 4x}{\sqrt{1 + x + 2x^2}} \, dx = \int u^{-1/2} \, du = 2u^{1/2} + C = 2 \sqrt{1 + x + 2x^2} + C
\]

Problem 23, §5.5, p395. Find \( \int \cos^4 x \sin x \, dx \).

Solution. Since \( \sin x \) is the negative of the derivative of \( \cos x \), we put \( u = \cos x \) with \( du = - \sin x \, dx \) or \( \sin x \, dx = -du \). This gives

\[
\int \cos^4 x \sin x \, dx = - \int u^4 \, du = -\frac{1}{5} u^5 + C = -\frac{1}{5} \cos^5 x + C
\]
Problem 24, §5.5, p395. Find $\int \frac{\cos(\frac{\pi}{x})}{x^2} \, dx$.

Solution. Rewrite the integrand as $\int \frac{1}{x^2} \cos(\frac{\pi}{x}) \, dx$ and remember that the derivative of $\frac{1}{x}$ is $-\frac{1}{x^2}$. This suggests the substitution $u = \frac{\pi}{x}$ with $du = -\frac{\pi}{x^2} \, dx$ and $\frac{dx}{x^2} = -\frac{du}{\pi}$. We find

$$\int \frac{1}{x^2} \cos(\frac{\pi}{x}) \, dx = -\frac{1}{\pi} \int \cos u \, du = -\frac{1}{\pi} \sin u + C = -\frac{1}{\pi} \sin(\frac{\pi}{x}) + C$$

Problem 26, §5.5, p395. Find $\int \cos x \cos(\sin x) \, dx$.

Solution. Since the $\cos x$ that appears at the front end of the integrand is the derivative of the inside function $\sin x$, choose $u = \sin x$, so $du = \cos x \, dx$. This gives

$$\int \cos x \cos(\sin x) \, dx = \int \cos u \, du = \sin u + C = \sin(\sin x) + C$$

Problem 39, §5.5, p395. Find $\int_{0}^{1} x^2(1 + 2x^3)^5 \, dx$.

Solution. This is very similar to a problem we did in class. Substitute $u = 1 + 2x^3$ which gives $du = 6x^2 \, dx$ and $x^2 \, dx = \frac{1}{6} \, du$. When $x = 0$, $u = 1$; and when $x = 1$, $u = 3$. Therefore

$$\int_{0}^{1} x^2(1 + 2x^3)^5 \, dx = \frac{1}{6} \int_{1}^{3} u^5 \, du = \left[ \frac{1}{6} u^6 \right]_{1}^{3} = \frac{1}{6} (3^6 - 1) = \frac{728}{36} = \frac{182}{9}$$
Problem 43, §5.5, p395. Find \( \int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx \).

Solution. The key to this is the fact that, except for a constant factor, the derivative of \( \sqrt{x} \) is \( \frac{1}{2\sqrt{x}} \). Set \( u = \sqrt{x} \), which gives \( du = \frac{1}{2\sqrt{x}} \, dx \) or \( \frac{dx}{\sqrt{x}} = 2 \, du \), with \( u = 2 \) when \( x = 4 \) and \( u = 1 \) when \( x = 1 \). Therefore
\[
\int_{1}^{4} \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx = 2 \int_{1}^{2} e^u \, du = [2e^u]_1^2 = 2(e^2 - e).
\]

Problem 51, §5.5, p395. Find \( \int_{e}^{e^4} \frac{dx}{x \sqrt{\ln x}} \).

Solution. Since \( \int \frac{dx}{x \sqrt{\ln x}} = \int (\ln x)^{-\frac{1}{2}} \frac{dx}{x} \) and \( \frac{1}{x} \) is the derivative of \( \ln x \), substitute \( u = \ln x \), which gives \( du = \frac{dx}{x} \). Then \( u = 1 \) when \( x = e \), and \( u = 4 \) when \( x = e^4 \), so
\[
\int_{e}^{e^4} \frac{dx}{x \sqrt{\ln x}} = \int_{1}^{4} u^{-\frac{1}{2}} \, du = [2\sqrt{u}]_1^4 = 4 - 2 = 2.
\]