Problem 8, §6.3, p471. Let $S$ be the surface of revolution that is obtained by revolving the curve $y = \frac{x^3}{6} + \frac{1}{2x}$, $\frac{1}{2} \leq x \leq 1$ about the $x$-axis. Find the surface area for $S$.

Solution. As was computed in Problem 8, p471, we have

$$1 + (y')^2 = 1 + \left(\frac{x^2}{2} - \frac{1}{2x^2}\right)^2 = 1 + \frac{x^4}{4} - \frac{1}{2} + \frac{1}{4x^4} = \frac{x^4}{4} + \frac{1}{2} + \frac{1}{4x^4} = \left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2$$

Therefore, the surface area, $A$, is given by

$$A = 2\pi \int_{\frac{1}{2}}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right)\sqrt{\left(\frac{x^2}{2} + \frac{1}{2x^2}\right)^2} \, dx$$

$$= 2\pi \int_{\frac{1}{2}}^{1} \left(\frac{x^3}{6} + \frac{1}{2x}\right) \left(\frac{x^2}{2} + \frac{1}{2x^2}\right) \, dx$$

$$= 2\pi \int_{\frac{1}{2}}^{1} \left(\frac{x^5}{12} + \frac{x}{4} + \frac{1}{4x^3}\right) \, dx$$

$$= 2\pi \left[ \frac{x^6}{72} + \frac{x^2}{3} + \frac{1}{8x^2} \right]_{\frac{1}{2}}^{1}$$

$$= \frac{2367\pi}{2304}$$

$$\approx 3.2275$$