Mathematics 120
Practice Test 2

Note: For this test, you may use the list of Frequently Used Maclaurin Series.

1. For each of the following limit statements (about a sequence), give the formal definition.

   (a) $\lim_{n\to\infty} a_n = L$ (L a real number)

   (b) $\lim_{n\to\infty} a_n = +\infty$

2. Do (a) or (b), not both. For either (a) or (b) give a proof showing that the given statement is true. Your proof should be written in complete sentences and should be one that is based on the relevant formal definition, not on limit theorems or known limits.

   (a) $\lim_{n\to\infty} \dfrac{7n-5}{n} = 7$

   (b) $\lim_{n\to\infty} (4n-2) = +\infty$

3. (a) Give the definition for "the series $\sum_{n=1}^{\infty} a_n$ is convergent". Write carefully, especially if you use the pesky word "it"!

   (b) Use the definition you gave in part (a), together with the Sandwich Theorem for sequences, to show that if $3 - \dfrac{1}{n} \leq a_1 + a_2 + \cdots + a_n \leq 3 + \dfrac{1}{n}$ for every n, then the series $\sum_{n=1}^{\infty} a_n$ converges. What is the value for $\sum_{n=1}^{\infty} a_n$?

4. If possible, give an example of the kind specified. If no such example exists, say so and briefly explain why.

   (a) a divergent nondecreasing sequence

   (b) a divergent p-series

   (c) a divergent series whose term sequence has limit 0

   (d) a convergent sequence $\{a_n\}$ and a convergent sequence $\{b_n\}$ for which the sequence $\{a_n + b_n\}$ diverges

5. Find the sum of the given series.

   (a) $\sum_{n=1}^{\infty} \dfrac{3^{n+2}}{4^n}$

   (b) $\sum_{n=1}^{\infty} \dfrac{6}{(6n-3)(6n+3)}$

   (c) $\sum_{k=0}^{\infty} (-1)^k \dfrac{(\pi)^{2k+1}}{(2k+1)!}$

6. For each of the series given below, decide whether the series converges. Show your reasoning.

   (a) $\displaystyle\sum_{n=1}^{\infty}\left(3+\frac{25}{n}\right)$

   (b) $\displaystyle\sum_{n=1}^{\infty}\frac{n\sqrt{n}+5}{n^2+3}$

   (c) $\displaystyle\sum_{n=3}^{\infty}\frac{1}{n(\ln n)^4}$

   (d) $\displaystyle\sum_{n=1}^{\infty}\frac{(6n+5)8^n}{5^n n!}$

7. For the given power series, find the interval of convergence. (In other words, find all values of x for which the series converges, including relevant endpoints, if any.)

$$\sum_{n=1}^{\infty}(-1)^n(2n)4^n x^n$$

8. Use Maclaurin series to find $\displaystyle\lim_{x\to 0}\frac{x\sin x}{\arctan(x^2)}$.

9. (a) Beginning with the Maclaurin series for $\dfrac{1}{1+x}$, find a series for $\dfrac{x}{1+x^5}$.

   (b) For which values of x can you be sure this series representation is valid and why?

   (c) Use the series you found in part (a) to show that

$$\begin{aligned}\int_0^{\frac{1}{2}}\frac{x}{1+x^5}\,dx &= \frac{1}{2^3}-\frac{1}{7\cdot 2^7}+\frac{1}{12\cdot 2^{12}}-\frac{1}{17\cdot 2^{17}}+\cdots\\ &\approx \frac{1}{2^3}-\frac{1}{7\cdot 2^7}+\frac{1}{12\cdot 2^{12}}-\frac{1}{17\cdot 2^{17}}\end{aligned}$$

   (d) What information does the Alternating Series Error Estimate give about the following approximation?

$$\int_0^{\frac{1}{2}}\frac{x}{1+x^5}\,dx \approx \frac{1}{2^3}-\frac{1}{7\cdot 2^7}+\frac{1}{12\cdot 2^{12}}$$