Mathematics 120 Solutions for Quiz 2
IP and u-substitution

1. (a) Find $\int 4x^3 \ln 8x \, dx$

Use integration by parts. Let $u = \ln 8x$ and $dv = 4x^3 dx$. Then $du = \frac{1}{x} \, dx$ and $v = x^4$. Then integration by parts and the power rule give

$$(1) \int 4x^3 \ln x \, dx = x^4 \ln 8x - \int x^3 \, dx = x^4 \ln x - \frac{x^4}{4} + C$$

(b) Find $\int x^2 \sin^6(x^3) \cos(x^3) \, dx$

Various possible substitutions or sequences of substitutions will work here. Or you might be able to guess and check without using a substitution. Here's one possible substitution. Let $w = \sin(x^3)$. This gives $dw = 3x^2 \cos(x^3) \, dx$ and so

$$\int x^2 \sin^6(x^3) \cos(x^3) \, dx = \frac{1}{3} \int w^6 \, dw = \frac{w^7}{21} + C = \frac{\sin^7(x^3)}{21} + C$$

(c) Find $\int_0^{2\pi} e^{2x} \cos x \, dx$

Use integration by parts, expecting a double-back. One can either work with definite integrals throughout or, instead, work with indefinite integrals until the original (indefinite) integral has been found, and then find the definite integral. The solution that follows uses definite integrals throughout.

I'll choose $u = e^{2x}$ and $dv = \cos x \, dx$. This gives $du = 2e^{2x} \, dx$ and $v = \sin x$, so

$$(1) \quad \int_0^{2\pi} e^{2x} \cos \, dx = e^{2x} \sin x \Big]_0^{2\pi} - 2 \int_0^{2\pi} e^{2x} \sin x \, dx = 2 \int_0^{2\pi} e^{2x} (-\sin x) \, dx$$

Now we apply integration by parts a second time. This time it's important to choose u and dv in a way that parallels the choice we made at first. With that in mind, and planning to keep the 2 in front, we choose $u = e^{2x}$ and $dv = -\sin x \, dx$. We have the same du as before, and $v = \cos x$. Thus the second integration by parts tells us that

$$(2)\quad 2\int_0^{2\pi} e^{2x}(-\sin x)\,dx = 2e^{2x}\cos x\Big]_0^{2\pi} - 4\int_0^{2\pi} e^{2x}\cos x\,dx$$

Combining (1) and (2) we have

$$5\int_0^{2\pi} e^{2x}\cos x\,dx = 2e^{2x}\cos x\Big]_0^{2\pi}$$

Evaluating the expression on the right and solving for the original integral we obtain

$$\int_0^{2\pi} e^{2x}\cos x\,dx = \frac{2}{5}[e^{4\pi} - 1]$$

2. Use integration by parts to verify the formula

$$\int \arcsin ax\,dx = x\arcsin ax + \frac{1}{a}\sqrt{1-a^2x^2} + C$$

Let $u = \arcsin ax$ and $dv = dx$. Then $du = \dfrac{a}{\sqrt{1-a^2x^2}}\,dx$ and $v = x$. The formula for integration by parts gives

$$\int \arcsin ax\,dx = x\arcsin ax - \int \frac{ax}{\sqrt{1-a^2x^2}}\,dx$$

Now let $w = 1 - a^2x^2$, and note that $dw = -2a^2x\,dx$ Therefore

$$-\int \frac{ax}{\sqrt{1-a^2x^2}}\,dx = \frac{1}{2a}\int w^{-\frac{1}{2}}\,dw = \frac{1}{a}w^{\frac{1}{2}} + C$$

Consequently

$$\int \arcsin ax\,dx = x\arcsin ax + \frac{1}{a}\sqrt{1-a^2x^2} + C$$

as was claimed.