Mathematics 120
Test 1

1. (20 points) Find three (3) of the following four (4) integrals.

   (a) $\int \mathrm{e}^x \cos^3(\mathrm{e}^x + 2) \sin(\mathrm{e}^x + 2)\, dx$

   (b) $\int \dfrac{x+3}{x^2+2x}\, dx$

   (c) $\int \arctan 2x dx$

   (d) $\int \mathrm{e}^{2x} \sin x\, dx.$

2. (30 points) Find the indicated limit (number, ∞, $-\infty$), if it exists. If the limit does not exist at all, say so and explain why. Show your reasoning. Calculator values or graphs are not enough.

   (a) $\lim\limits_{x\to 0} \dfrac{\sin x - \mathrm{e}^x + 1}{\cos x - 1}$

   (b) $\lim\limits_{x\to 1^+} \dfrac{4+x}{x-1}$

   (c) $\lim\limits_{x\to 1} \big(f(x)(x-1)^2\big)$, given that $f(x) = \begin{cases} 4x & \text{if } x > 1 \\ 5 & \text{if } x < 1 \end{cases}$

   (d) $\lim\limits_{x\to 0} \big(\cos x + 4x\big)^{\frac{1}{x}}$

   (e) $\lim\limits_{x\to\infty} x \sin(\dfrac{1}{x})$

3. (15 points) Evaluate the convergent improper integral $\displaystyle\int_0^{+\infty} x\mathrm{e}^{-x}\, dx.$

4. (10 points) The improper integral $\displaystyle\int_{-2}^{1} \frac{1}{x^2}\, dx$ is divergent. Show why.

5. (10 points) For the function $f(x) = x^2 + \sin(\pi x)$, find the Taylor polynomial of order 3 with basepoint $a = 1$.

6. (15 points) For the function $f(x) = \mathrm{e}^{x-1}$ and the basepoint $a = 1$ find the Taylor series, including the general term, in two different ways:

   (a) by using the definition/formula for Taylor series

   (b) by making an appropriate modification to the known Maclaurin series for e^x