1. To convert from rectangular or Cartesian coordinates \((x, y)\) to polar coordinates \([r, \theta]\), use
\[
r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}
\]
to find \(r\) and \(\theta\), choosing \(\theta\) to satisfy \(0 \leq \theta \leq 2\pi\) and to match the quadrant where \((x, y)\) is located. In the other direction, to convert from polar coordinates \([r, \theta]\) to rectangular coordinates \((x, y)\), use
\[
x = r \cos \theta, \quad y = r \sin \theta
\]
to find \(x\) and \(y\).

2. To convert from rectangular or Cartesian coordinates \((x, y, z)\) to cylindrical coordinates \([r, \theta, z]\), as for polar coordinates, use
\[
r = \sqrt{x^2 + y^2}, \quad \tan \theta = \frac{y}{x}
\]
to find \(r\) and \(\theta\), choosing \(\theta\) to satisfy \(0 \leq \theta \leq 2\pi\) and to match the quadrant in the \(xy\)-plane where \((x, y)\) would be located. Keep \(z\) as is. Note that cylindrical coordinates are simply polar coordinates with \(z\) thrown in. So one converts to cylindrical coordinates just as one would convert to polar coordinates, leaving \(z\) unchanged.

3. To convert from cylindrical coordinates \([r, \theta, z]\) to rectangular coordinates \((x, y, z)\), as for polar coordinates, use
\[
x = r \cos \theta, \quad y = r \sin \theta
\]
to find \(x\) and \(y\). Keep \(z\) unchanged.

4. To convert from rectangular coordinates \((x, y, z)\) to spherical coordinates \([\rho, \theta, \phi]\), use
\[
\rho = \sqrt{x^2 + y^2 + z^2}, \quad \tan \theta = \frac{y}{x}, \quad \cos \phi = \frac{z}{\rho}
\]
to find \(\rho, \theta,\) and \(\phi\), choosing \(\theta\) to satisfy \(0 \leq \theta \leq 2\pi\) and to match the quadrant in the \(xy\)-plane where \((x, y)\) would be located, and choosing \(\phi\) to satisfy \(0 \leq \phi \leq \pi\).
5. To convert from spherical coordinates \( \{ \rho, \theta, \phi \} \) to rectangular coordinates \( (x, y, z) \), use

\[
x = \rho \sin \phi \cos \theta, \quad y = \rho \sin \phi \sin \theta, \quad z = \rho \cos \phi.
\]

Note that the formulas for \( x \) and \( y \) are easier to remember if you think of them as \( x = r \cos \theta \) and \( y = r \sin \theta \), with \( r \) replaced by \( \rho \sin \phi \).

6. To convert from cylindrical coordinates \( [r, \theta, z] \) to spherical coordinates \( \{ \rho, \theta, \phi \} \), use

\[
\rho = \sqrt{r^2 + z^2}
\]

to find \( \rho \). Note how this agrees with \( \rho = \sqrt{x^2 + y^2 + z^2} \). Keep \( \theta \) unchanged. Use

\[
\cos \phi = \frac{z}{\rho} = \frac{z}{\sqrt{r^2 + z^2}}
\]

to find \( \phi \), choosing \( \phi \) to satisfy \( 0 \leq \phi \leq \pi \).

7. To convert from spherical coordinates \( \{ \rho, \theta, \phi \} \) to cylindrical coordinates \( [r, \theta, z] \), use

\[
r = \rho \sin \phi, \quad z = \rho \cos \phi
\]

to find \( r \) and \( z \), and keep \( \theta \) unchanged.