Mathematics 205
HWK 18a Solutions
Section 16.1 p741

Problem 13, §16.1, p741. The table in the text gives values of $f(x, y)$, the number of milligrams of mosquito larvae per square meter in a swamp. If x and y are in meters and R is the rectangle $0 \leq x \leq 8,\ 0 \leq y \leq 6$, estimate $\int_R f(x, y)\, dA$. give units and interpret your answer. Here, in list form, are the values given in the table.

For $y = 0$, the values for $x = 0, 4, 8$, respectively, are 1, 3, 6. (This is the first row of data in the table.)
For $y = 3$, the values for $x = 0, 4, 8$, are 2, 5, 9.
For $y = 6$, the values for $x = 0, 4, 8$, are 4, 9, 15.

Solution. There are lots of possibilities here. One would be to divide the given rectangle into 4 congruent subrectangles, each of area 12. Then we could form either an upper Riemann sum, by using the highest function value given for each subrectangle, or a lower Riemann sum, using the lowest function value given for each subrectangle. Or we could use the value from the northeast corner each time; or the southeast corner; etc. Draw yourself a picture of the rectangle and its subrectangles, like the ones we drew in class, to make it easier to pull out the numbers you want. Here are the calculations for several possible estimates that use 4 congruent subrectangles:

$$\text{upper sum} = (5 + 9 + 15 + 9)(12) = 456$$

$$\text{lower sum} = (1 + 2 + 5 + 3)(12) = 132$$

$$\text{average of upper and lower sums} = \frac{1}{2}(456 + 132) = 294$$

$$\text{northeast sum} = (9 + 15 + 5 + 9)(12) = 436$$

$$\text{southeast sum} = (5 + 9 + 3 + 6)(12) = 276$$

$$\text{southwest sum} = (1 + 3 + 2 + 5)(12) = 132$$

$$\text{northwest sum} = (9 + 15 + 5 + 9)(12) = 436$$

Here's yet another possibility. Divide the rectangle into 16 congruent subrectangles, each of area 3. For each of these subrectangles we have one function value we can use. Use the resulting Riemann sum, which would be (using subrectangles from left to right on the bottom row, then left to right on the second row up, and so forth)

$$(1 + 3 + 3 + 6 + 2 + 5 + 5 + 9 + 2 + 5 + 5 + 9 + 4 + 9 + 9 + 15)(3) = 279.$$

I'll leave it to you to pick out what you think is the best estimate, whether from among these or from among some larger collection of estimates you could find! Or perhaps you'd like to pick some of these and average them. Just be sure to specify what you're doing.

P.S. I did all this picking off values and adding things up hurriedly in my head, so there may well be an error or two!

The units, by the way, will be number of milligrams of mosquito larvae, with the integral representing the milligram-amount of larvae in the rectangle of swamp.

Problem 15, §16.1, p741. Let B be the bottom half of the region inside the unit circle centered at the origin. Decide (without calculation) whether $\int_B dA$ (i.e. $\int_B 1\,dA$) is positive, negative, or zero.

Solution. Every Riemann sum for this integral would be approximately equal to the area of B, which is positive. The value for the integral is exactly the area of B, a positive number.

Problem 16, §16.1, p741. Let R be the right half of the region inside the unit circle centered at the origin. Decide (without calculation) whether $\int_R 5x\,dA$ is positive, negative, or zero.

Solution. The integrand $5x$ is positive on R (except at those few points that are on the boundary of R, where it is zero). The Riemann sums for this integral will each be positive, and if we think of $5x$ as a height function, then the Riemann sums will approach the numerical value for a particular volume, which will be positive. So the specified integral is positive.

Problem 17, §16.1, p741. Let B be the bottom half of the region inside the unit circle centered at the origin. Decide (without calculation) whether $\int_B 5x\,dA$ is positive, negative, or zero.

Solution. The region B is symmetric with respect to the y-axis. At points that are mirror images of each other, relative to the y-axis, the integrand, $5x$, takes values that have the same absolute value but opposite signs. The integral over the right half of B will be positive, the integral over the left half of B will be negative, and these two integrals will exactly cancel each other out, so the integral over B will be zero.