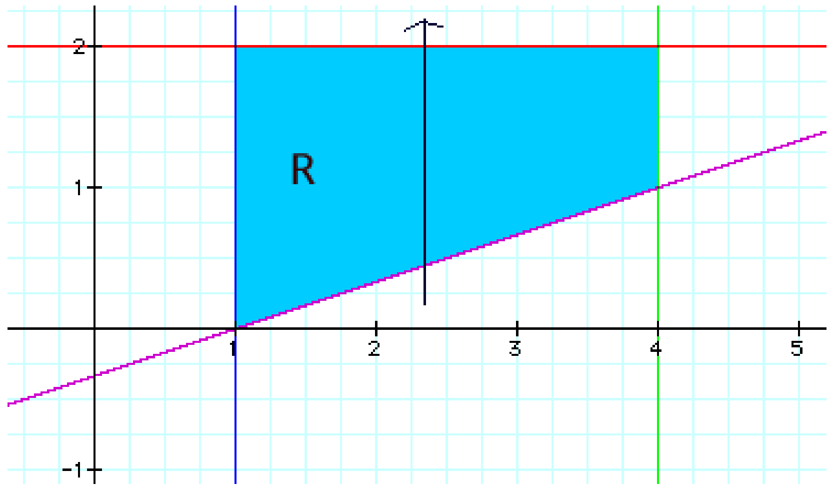
Mathematics 205
HWK 18b Solutions
Section 16.2 p750

Problem 9, §16.2, p750. For the region R as shown, write $\int_R f\,dA$ as an iterated integral.



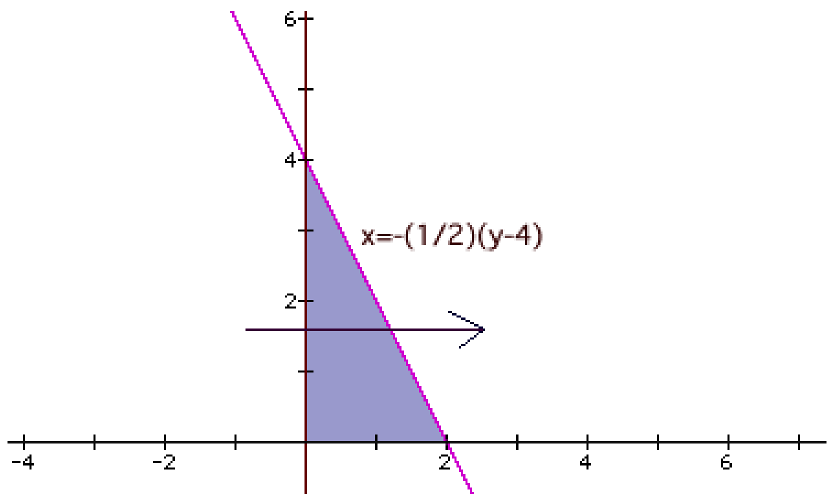
Solution. Note that the equation for the line that determines the bottom edge of R is $y = \frac{1}{3}(x-1)$. It will be easier to integrate with respect to y first (and thus shoot vertical arrows to determine the inside limits of integration). Shooting a vertical arrow, as shown, through the region R, we see that the arrow enters the region when $y = \frac{1}{3}(x-1)$ and leaves the region when $y = 2$. So the inner integral will be over the vertical interval $[\frac{1}{3}(x-1), 2]$. For these intervals to sweep out R, we need to have x vary from $x = 1$ to $x = 4$. Or, putting it differently, the leftmost arrow that actually hits the region would be the arrow for $x = 1$. Similarly, the rightmost arrow that hits R would be for $x = 4$. Thus the interval of integration for the outer integral will be $[1, 4]$. In summary, we have

$$\int_R f\,dA = \int_1^4 \int_{\frac{1}{3}(x-1)}^2 f(x,y)\,dy\,dx.$$

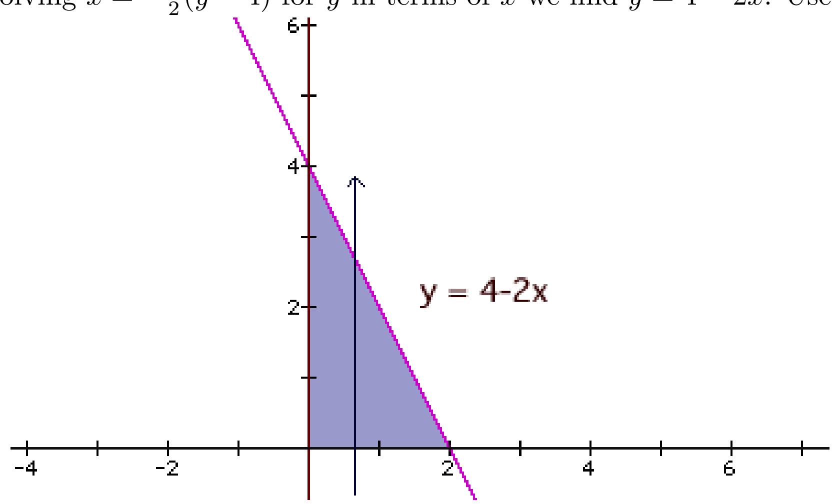
Note that, as always, the limits of integration only depend on the region R that we are integrating over. They do not depend on the function f that is being integrated.

Problem 25, §16.2, p750. Consider the integral $\int_0^4 \int_0^{-(y-4)/2} g(x,y)\, dx\, dy$.
(a) Sketch the region over which the integration is being performed.
(b) Write the integral with the order of the integration reversed.

Solution. (a) The inner integral is with respect to x and x varies from 0 to $-\frac{1}{2}(y-4)$. This tells us that a horizontal arrow through the region hits the region at $x=0$ and leaves at $x=-\frac{1}{2}(y-4)$. So the line $x=0$, i.e. the y-axis, will form the left edge of the region, while the line $x=-\frac{1}{2}(y-4)$, or $2x=-y+4$, will form the right edge of the region. Sketch this much, notice that the line $2x=-y+4$ hits the y-axis exactly at $y=4$. So the region will have a topmost point at $y=4$ on the $y-axis$. Finally, the outer integral tells us that the bottom edge of the region is formed by the x-axis, where $y=0$. Putting the pieces together gives us the sketch (shown with a horizontal arrow):



(b) To reverse the order of integration, we use the sketch of the region but begin by shooting a vertical arrow. We will also need to rewrite the equation of the top side in order to have y as a function of x. Solving $x=-\frac{1}{2}(y-4)$ for y in terms of x we find $y=4-2x$. Use a new sketch:



With x fixed, a vertical arrow enters the region where $y = 0$ and leaves it where $y = 4 - 2x$. We need to use vertical arrows for x between 0 and 2. Thus the reversed iterated integral is

$$\int_0^2 \int_0^{4-2x} g(x, y)\, dy\, dx.$$