Problem 9, §16.2, p750. For the region $R$ as shown, write $\int_R f \, dA$ as an iterated integral.

Solution. Note that the equation for the line that determines the bottom edge of $R$ is $y = \frac{1}{3}(x-1)$. It will be easier to integrate with respect to $y$ first (and thus shoot vertical arrows to determine the inside limits of integration). Shooting a vertical arrow, as shown, through the region $R$, we see that the arrow enters the region when $y = \frac{1}{3}(x-1)$ and leaves the region when $y = 2$. So the inner integral will be over the vertical interval $[\frac{1}{3}(x-1), 2]$. For these intervals to sweep out $R$, we need to have $x$ vary from $x = 1$ to $x = 4$. Or, putting it differently, the leftmost arrow that actually hits the region would be the arrow for $x = 1$. Similarly, the rightmost arrow that hits $R$ would be for $x = 4$. Thus the interval of integration for the outer integral will be [1, 4]. In summary, we have

$$\int_R f \, dA = \int_1^4 \int_{\frac{1}{3}(x-1)}^2 f(x, y) \, dy \, dx.$$ 

Note that, as always, the limits of integration only depend on the region $R$ that we are integrating over. They do not depend on the function $f$ that is being integrated.
Problem 25, §16.2, p750. Consider the integral $\int_0^4 \int_0^{-(y-4)/2} g(x, y) \, dx \, dy$.

(a) Sketch the region over which the integration is being performed.
(b) Write the integral with the order of the integration reversed.

Solution. (a) The inner integral is with respect to $x$ and $x$ varies from 0 to $-\frac{1}{2}(y - 4)$. This tells us that a horizontal arrow through the region hits the region at $x = 0$ and leaves at $x = -\frac{1}{2}(y - 4)$. So the line $x = 0$, i.e. the $y$-axis, will form the left edge of the region, while the line $x = -\frac{1}{2}(y - 4)$, or $2x = -y + 4$, will form the right edge of the region. Sketch this much, notice that the line $2x = -y + 4$ hits the $y$-axis exactly at $y = 4$. So the region will have a topmost point at $y = 4$ on the $y$–axis. Finally, the outer integral tells us that the bottom edge of the region is formed by the $x$-axis, where $y = 0$. Putting the pieces together gives us the sketch (shown with a horizontal arrow):

(b) To reverse the order of integration, we use the sketch of the region but begin by shooting a vertical arrow. We will also need to rewrite the equation of the top side in order to have $y$ as a function of $x$. Solving $x = -\frac{1}{2}(y - 4)$ for $y$ in terms of $x$ we find $y = 4 - 2x$. Use a new sketch:
With $x$ fixed, a vertical arrow enters the region where $y = 0$ and leaves it where $y = 4 - 2x$. We need to use vertical arrows for $x$ between 0 and 2. Thus the reversed iterated integral is

$$\int_0^2 \int_0^{4-2x} g(x, y) \, dy \, dx.$$