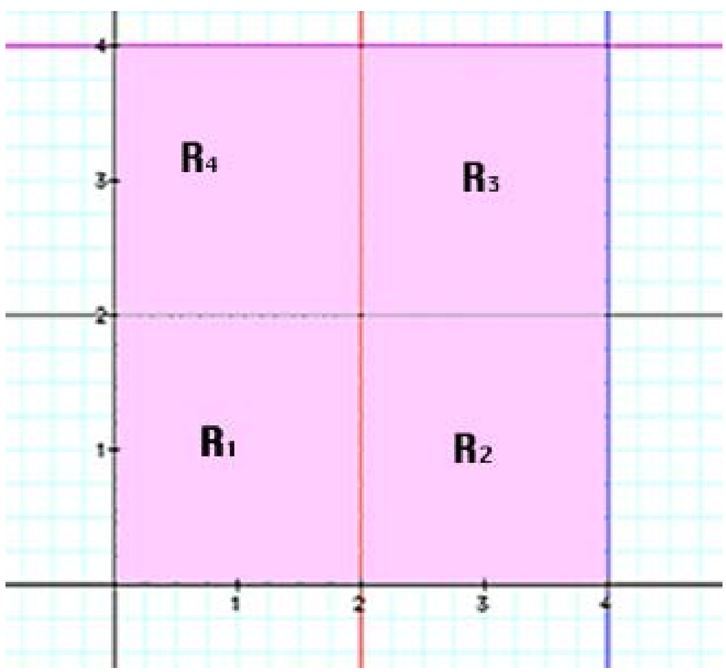
Mathematics 205
HWK 19a Solutions
Section 16.1 p741

Problem 3, §16.1, p741. Let R be the rectangle with vertices $(0,0),(4,0),(4,4)$, and $(0,4)$ and let $f(x,y)=\sqrt{xy}$.

(a) Find reasonable upper and lower bounds for $\int_R f\,dA$ without subdividing R.

(b) Estimate $\int_R f\,dA$ by partitioning R into four subrectangles and evaluating f at its maximum and minimum values on each subrectangle.

Solution. (a) The minimum value for f on R is zero, and the maximum value is $\sqrt{4\cdot 4}=4$. Moreover, the area of R (which is a square with side-length 4) is 16. Therefore the integral certainly lies between 0 and $4(16)=64$.
(b) The text uses partitions that have a uniform Δx and a uniform Δy (so one for which all the subrectangles are congruent). So in this case each subrectangle will be 2 units by 2 units. I'll call the subrectangles R_1, R_2, R_3, R_4, labeled as in the diagram shown below.



For each subrectangle, the minimum value for f will occur at its lower left-hand corner, and the maximum value for f will occur at its upper right-hand corner. So the approximating sum that uses minimum values on each subrectangle would be

$$0(4)+0(4)+0(4)+2(4)=8.$$

With maximum values on each subrectangle, the approximating sum would be

$$2(4)+\sqrt{8}(4)+\sqrt{8}(4)+4(4)=24+8\sqrt{8}=24+16\sqrt{2}\approx 46.6274.$$

In other words

$$8 \leq \int_R f\,dA \leq 46.6274.$$

As the answer in the text points out, the average of the upper sum and the lower sum, which is very likely a better estimate than either of them, would be $\frac{1}{2}(32 + 16\sqrt{2}) \approx 27.3137$.

Problem 11, §16.1, p741. A biologist studying insect populations measures the population density of flies and mosquitos at various points in a rectangular study region. The graphs of the two population densities for the region are show in the text. Assuming that the units along the corresponding axes are the same in the two graphs, are there more flies or more mosquitos in the region?

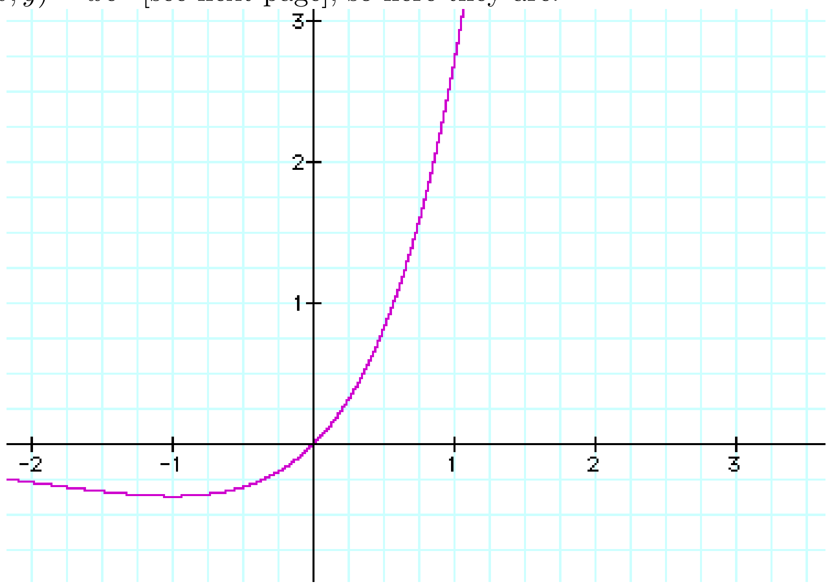
Solution. To get the total amount of flies in the region (in whatever units are being used here – actual count, it appears) we would integrate the fly-density function, which is the one shown in the top graph. Numerically, this will be the same as the volume under this graph and above the rectangular region in question. This volume appears to be considerably smaller than the corresponding volume for the mosquito-density function. So there are more mosquitos than flies.

Problem 24, §16.1, p741. If D denotes the region inside the unit circle centered at the origin, decide whether the integral $\int_D \sin y\,dA$ is positive, negative, or zero.

Solution. The region D is symmetric relative to the x-axis [and, for that matter, relative to the y-axis], and f is an odd function of y [that is, for each (x, y) in D, $f(x, -y) = -f(x, y)$. Therefore the integral over the top half of D will cancel the integral over the bottom half of D. [In fact, since the radius of the circle is less than $\frac{\pi}{2}$, the integral over the top half is positive and the integral over the bottom half is exactly the opposite of that.] So $\int_D \sin y\,dA = 0$.

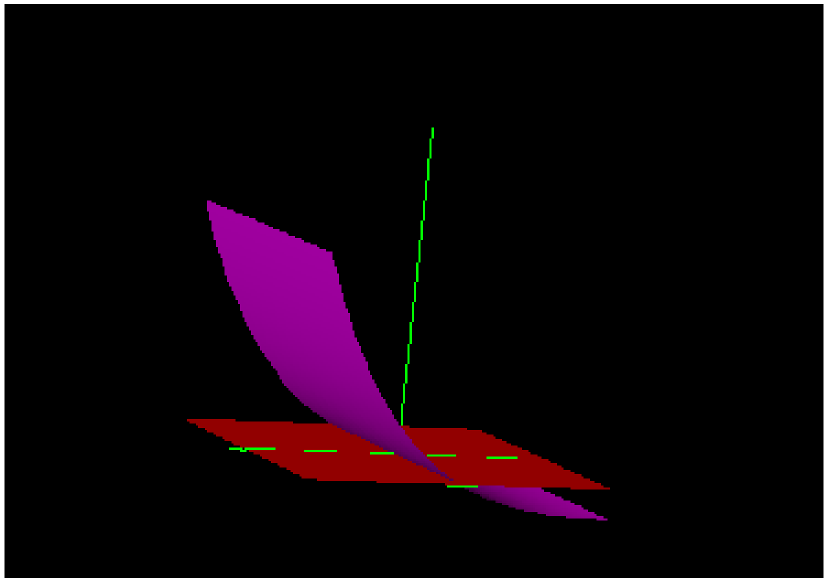
Problem 27, §16.1, p741. If D denotes the region inside the unit circle centered at the origin, decide whether the integral $\int_D x\mathrm{e}^x \, dA$ is positive, negative, or zero.

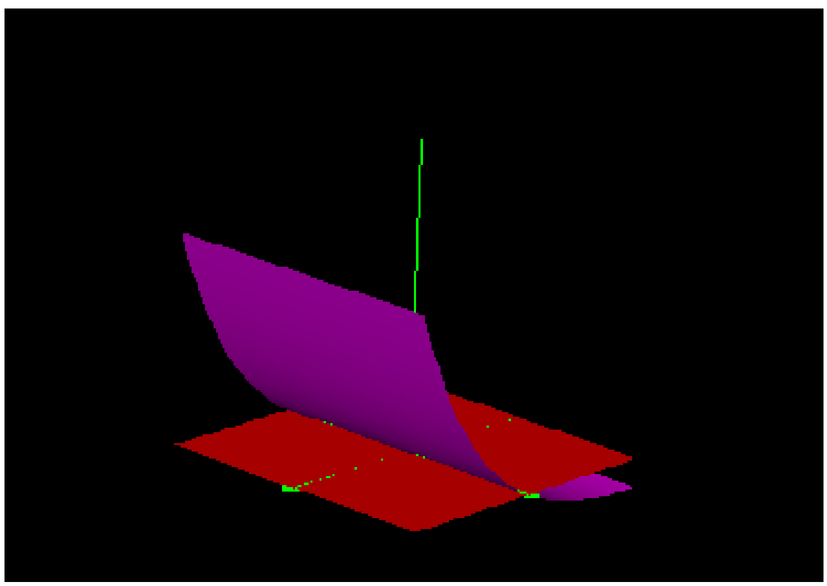
Solution. Although the integrand is positive on the right half of D and negative on the left half of D, the integrand does not have the necessary symmetry to let us conclude that the integral is zero. That is, we do not have $f(-x, y) = -f(x, y)$. In fact, for $x > 0$, $|x\mathrm{e}^x| > |(-x)\mathrm{e}^{(-x)}|$. Therefore the positive contribution from the integral of $x\mathrm{e}^x$ over the right half of D will dominate the negative contribution from the integral of the same function over the left half of D. Therefore $\int_D x\mathrm{e}^x \, dA > 0$. It might be interesting here to see a graph of $f(x) = x\mathrm{e}^x$ [shown below; as you can easily verify, it passes through the origin and there is a relative minimum where $x = -1$] and/or a graph of $z = f(x, y) = x\mathrm{e}^x$ [see next page], so here they are.



Graph for $f(x) = x\mathrm{e}^x$

For the two surface graphs on the next page, when you're looking at these in color, the flat red surface is just the plane $z = 0$, which I included for reference. The scarlet surface is $z = f(x, y) = x\mathrm{e}^x$. The two graphs use two slightly different points of view. The surface graph for $z = f(x, y)$ is exactly what you would expect after looking at the related function graph: we're graphing a surface whose equation has one variable missing. What I didn't attempt to show here is the region of integration, which is the region inside the unit circle in the xy-plane [i.e. inside the red plane $z = 0$]. But you can see how over half of the disc we would be integrating a positive function that grows pretty quickly, while over the other half we would be integrating a much smaller negative function.





Graphs for $z = f(x, y) = xe^x$