For many of these problems, it may be helpful to make a sketch or a schematic diagram. In fact, it’s probably a good idea to get in the habit of doing this.

**Problem 5, §1.1, p22.** Given \( \mathbf{v} = (2, 1) \) and \( \mathbf{w} = (1, 2) \), sketch the vectors \( \mathbf{v}, \mathbf{w}, -\mathbf{v}, \mathbf{v} + \mathbf{w}, \) and \( \mathbf{v} - \mathbf{w} \).

See text, p555.

**Problem 9, §1.1, p22.** What restrictions must be made on \( x, y, \) and \( z \) in order that \( (x, y, z) \) represents a point on the \( y \)-axis? on the \( z \)-axis? in the \( xz \)-plane? in the \( yz \)-plane?

**Solution.**

For \( (x, y, z) \) to be on the \( y \)-axis: \( x = 0 \) and \( z = 0 \).

For \( (x, y, z) \) to be on the \( z \)-axis: \( x = 0 \) and \( y = 0 \).

For \( (x, y, z) \) to be on the \( xz \)-plane: \( y = 0 \).

For \( (x, y, z) \) to be on the \( yz \)-plane: \( x = 0 \).

**Problem 13, §1.1, p22.** Find a parametric equation for the line passing through the point \((-1, -1, -1)\) in the direction of the vector \( \mathbf{j} \).

**Solution.** Using the point-direction form for a line, we get the equation \( \ell(t) = (-1, -1, -1) + t\mathbf{j}, -\infty < t < \infty \), which can also be written as \( \ell(t) = (-1, -1 + t, -1), -\infty < t < \infty \) or as \( \ell(t) = -\mathbf{i} + (t - 1)\mathbf{j} - \mathbf{k}, -\infty < t < \infty \). Other correct answers are possible.

**Problem 15, §1.1, p22.** Find a parametric equation for the line passing through the points \((-1, -1, -1)\) and \((1, -1, 2)\).

**Solution.** Here’s one solution, using point-direction form for a line. (If you use point-point form directly, you’ll get the same result, perhaps in slightly different notation.) With \( P \) as the point \((-1, -1, -1)\), \( Q \) as the point \((1, -1, 2)\), and \( \mathbf{v} \) as the vector \( \overrightarrow{PQ} \), the line in question is the same as \( \ell(t) = \overrightarrow{OP} + t\mathbf{v}, -\infty < t < \infty \). Since \( \mathbf{v} = \overrightarrow{PQ} = 2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k} \), this line is therefore

\[
\ell(t) = (-1, -1, -1) + t(2, 0, 3), -\infty < t < \infty
\]

If you prefer, you could write this as

\[
\ell(t) = (-1 + 2t, -1, -1 + 3t), \infty < t < \infty
\]

or as

\[
x = -1 + 2t, y = -1, z = -1 + 3t, -\infty < t < \infty
\]
or as
\[ \ell(t) = (2t - 1)i - j + (3t - 1)k, \quad -\infty < t < \infty \]

Other correct answers are also possible.

**Problem 27, §1.1, p22.** Write the chemical equation \( CO + H_2O = H_2 + CO_2 \) as an equation in ordered triples \((x_1, x_2, x_3)\) where \(x_1, x_2, x_3\) are the number of carbon, hydrogen, and oxygen atoms, respectively, in each molecule.

**Solution.** Writing \( CO = (1, 0, 1) \), \( H_2O = (0, 2, 1) \), \( H_2 = (0, 2, 0) \), and \( CO_2 = (1, 0, 2) \), the equation becomes
\[
(1, 0, 1) + (0, 2, 1) = (0, 2, 0) + (1, 0, 2)
\]

Since this can be simplified to read \((1, 2, 2) = (1, 2, 2)\), the chemical equation is balanced.

**Problem 28, §1.1, p22.**

(a). Write the chemical equation \( pC_3H_4O_3 + qO_2 = rCO_2 + sH_2O \) as an equation in ordered triples with unknown coefficients \( p, q, r \), and \( s \).

(b). Find the smallest positive integer solution for \( p, q, r \), and \( s \).

**Solution.**

(a). Replace \( C_3H_4O_3 \) by the triple \((3, 4, 3)\), \( O_2 \) by \((0, 0, 2)\), \( CO_2 \) by \((1, 0, 2)\), and \( H_2O \) by \((0, 2, 1)\). With these replacements the given chemical equation becomes
\[
p(3, 4, 3) + q(0, 0, 2) = r(1, 0, 2) + s(0, 2, 1)
\]

or
\[
(3p, 4p, 3p + 2q) = (r, 2s, 2r + s)
\]

(b). Equating the individual entries and simplifying a little gives \( 3p = r, 2p = s \), and \( 3p + 2q = 2r + s \). Since \( p, q, r \), and \( s \) must be whole numbers that are at least 1, and since both \( r \) and \( s \) are integral multiples of \( p \), we try setting \( p = 1 \). This gives \( r = 3, s = 2 \), and then \( 3 + 2q = 8 \) so \( 2q = 5 \). This won’t work, since \( q \) must be a whole number. So set \( p = 2 \) and get \( r = 6, s = 4 \), and \( 6 + 2q = 16 \) or \( 2q = 10, q = 5 \). This works, so it appears that the smallest solution using positive integers is \( p = 2, r = 6, s = 4, q = 5 \). In fact, the equations we’re working with could be reduced to \( r = 3p, s = 2p, q = \frac{5}{2}p \), which lets us see for sure that setting \( p = 2 \) gives the smallest possibility.