**Problem 11, §1.1, p11.** Describe the points that lie in the plane spanned by the vectors $v_1 = (2, 7, 0)$ and $v_2 = (0, 2, 7)$.

**Solution.** If we take all the scalar multiples of $v_1$, we get a line through the origin with direction $v_1$. Similarly, taking all the scalar multiples of $v_2$ gives a line through the origin with direction $v_2$. Adding a scalar multiple of $v_1$ to a scalar multiple of $v_2$ will give a vector lying in the plane determined or spanned by $v_1$ and $v_2$. So the plane in question can be described as the set of points that are endpoints of vectors of the form $s v_1 + t v_2$ for some real number $s$ and some real number $t$. In other words, the points in the specified plane are the points $(2s, 7s + 2t, 7t), -\infty < s < \infty, -\infty < t < \infty$. Other correct answers are possible.

**Problem 18, §1.1, p22.** Find the points of intersection of the line $x = 3 + 2t, y = 7 + 8t, z = \frac{-3}{2} + t$ with the coordinate planes. In other words, find the points of intersection of the line $\ell(t) = (3 + 2t, 7 + 8t, \frac{-3}{2} + t)$ with the coordinate planes.

**Solution.** Intersection with the $xy$-plane. Points on the $xy$-plane are characterized by the condition $z = 0$, so here’s the plan:
We’ll set $z = 0$ in the equation of the line and use that to solve for $t$.
Then we’ll use the resulting value of $t$ to find the $x$- and $y$-coordinates of the point we want.
Now here are the results.
Setting $z = \frac{-3}{2} + t = 0$ gives $t = 2$ as the $t$-value for the desired point of intersection. Then putting $t = 2$ gives $x = (3 + 2t)|_{t=2} = 3 + 4 = 7$ and $y = (7 + 8t)|_{t=2} = 7 + 16 = 23$.
The desired point of intersection is $(7, 23, 0)$.

Intersection with the $yz$-plane. Setting $x = 0$ gives $3 + 2t = 0$ so $t = -\frac{3}{2}$. For this value of $t$, we find $y = 7 - \frac{24}{2} = -5$ and $z = -2 - \frac{3}{2} = -\frac{7}{2}$. The point of intersection is $(0, -5, -\frac{7}{2})$.

Intersection with the $xz$-plane. Setting $y = 0$ gives $7 + 8t = 0$ so $t = -\frac{7}{8}$. This value of $t$ yields $x = 3 - \frac{14}{8} = \frac{5}{4}$ and $z = -2 - \frac{7}{8} = -\frac{23}{8}$, so the point of intersection is $(\frac{5}{4}, 0, -\frac{23}{8})$.

**Problem 6, §1.2, p36.** Given $u = 15i - 2j + 4k$ and $v = \pi i + 3j - k$, compute $||u||$, $||v||$, and $u \cdot v$.

**Solution.** Since $||u||$ is the same as the square root of the dot product of $u$ with itself, we find that

$$||u|| = \sqrt{(15)^2 + (-2)^2 + 4^2} = \sqrt{245}$$
Similarly, \[ ||v|| = \sqrt{\pi^2 + 3^2 + (-1)^2} = \sqrt{\pi^2 + 10} \]

Finally, we find the dot product of \( \mathbf{u} \) and \( \mathbf{v} \) by multiplying corresponding entries and then adding:

\[ \mathbf{u} \cdot \mathbf{v} = (15)(\pi) + (-2)(3) + (4)(-1) = 15\pi - 10 \]

**Problem 12, §1.2, p36 (for Problem 6).** Normalize the vector \( \mathbf{u} \) from Exercise 6. In other words, normalize the vector \( \mathbf{u} = (15, -2, 4) \).

**Solution.** To normalize a given vector means to find a unit vector with the same direction as the given vector. Since \( \mathbf{u} \) was found to have length \( \sqrt{245} \), we need to shorten \( \mathbf{u} \) by a factor of \( \frac{1}{\sqrt{245}} \). In other words, the unit vector we’re looking for is \( \frac{1}{\sqrt{245}} \mathbf{u} \) or \( \frac{1}{\sqrt{245}}(15\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}) \).

**Problem 14, §1.2, p36.** Given \( \mathbf{u} = -\mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{v} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} \), find the orthogonal projection of \( \mathbf{u} \) onto \( \mathbf{v} \).

**Solution.** Denoting the specified projection as \( \text{proj}_\mathbf{v} \mathbf{u} \), we have

\[
\text{proj}_\mathbf{v} \mathbf{u} = \frac{\mathbf{v} \cdot \mathbf{u}}{||\mathbf{v}||^2} \mathbf{v}
\]

Using the specified values for \( \mathbf{u} \) and \( \mathbf{v} \), we have \( \mathbf{v} \cdot \mathbf{u} = (-1)(2) + (1)(1) + (1)(-3) = -4 \) and \( ||\mathbf{v}||^2 = 2^2 + 1^2 + (-3)^2 = 14 \). Therefore

\[
\text{proj}_\mathbf{v} \mathbf{u} = \frac{-4}{14} \mathbf{v} = -\frac{2}{7}(2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) = -\frac{1}{7}(4\mathbf{i} + 2\mathbf{j} - 6\mathbf{k})
\]

**Problem 16, §1.2, p36.** What restrictions must be made on \( b \) so that the vector \( 2\mathbf{i} + b\mathbf{j} \) is orthogonal to (a) \(-3\mathbf{i} + 2\mathbf{j}\)? (b) \( \mathbf{k} \)?

**Solution.** (a) Two nonzero vectors are orthogonal if and only if their dot product is zero. So we need to have \( (2\mathbf{i} + b\mathbf{j}) \cdot (-3\mathbf{i} + 2\mathbf{j}) = 0 \) or, in other words, \( (2)(-3) + (b)(2) + (0)(0) = 0 \). This gives \( 2b = 6 \) or \( b = 3 \).

(b) In this case we need \( (2)(0) + (b)(0) + (0)(1) = 0 \) or simply \( 0 = 0 \). Since this is true irrespective of what \( b \) is, there is no restriction on \( b \), a result we might also have noticed by inspection, since the vector \( \mathbf{k} \) is orthogonal to every vector that lies in the \( xy \)-plane, as each \( 2\mathbf{i} + b\mathbf{j} \) does.

**Problem 17, §1.2, p36.** Find two nonparallel vectors, both orthogonal to \( (1, 1, 1) \).
Solution. There are lots of possibilities here. For the orthogonality, we need vectors \((x, y, z)\) that give a dot product with \((1, 1, 1)\) of 0. In other words we need to make sure that \(x + y + z = 0\). To guarantee that the two vectors we use are not parallel, we need to make sure that neither one is a scalar multiple of the other. So here’s one possible correct answer for this problem: \((1, -1, 0), (2, 1, -1)\). Lots of other correct answers are possible.

Problem 1, §1.5, p73. Calculate the dot product of \(x = (1, -1, 0, 2) \in \mathbb{R}^4\) and \(y = (1, 2, 3, 4) \in \mathbb{R}^4\).

Solution. \(x \cdot y = (1)(1) + (-1)(2) + (0)(3) + (2)(4) = 1 - 2 + 8 = 7\)

Problem A. Suppose \(u\) and \(v\) are vectors (in the same dimensional space) for which \(u\) has length 8, \(v\) has length 2, and the angle between \(u\) and \(v\) is \(\frac{\pi}{3}\). Find the inner product of \(u\) and \(v\).

Solution. From the relation between inner product and angle, and from the given information about \(u\) and \(v\), we have \(u \cdot v = ||u|| \cdot ||v|| \cdot \cos \frac{\pi}{3} = 8(2)\frac{1}{2} = 8\).

Problem B. Suppose \(u, v,\) and \(w\) are three vectors (in the same dimensional space) for which the dot product of \(u\) and \(v\) is the same as the dot product of \(u\) and \(w\). Must \(v = w\)? (In other words, can we cancel the \(u\)?)

Solution. Definitely not, even if the the vector \(u\) is a nonzero vector. You should be able to make up some examples to show this. For instance, in \(\mathbb{R}^3\), we could use \(u = (1, 1, 1), v = (1, 1, 0)\), and \(w = (0, 1, 1)\). For this example we would have \(u \cdot v = u \cdot w\) but \(v \neq w\).