Mathematics 205
HWK 2 Solutions
Section 12.4 p588

Problem 3, §12.4, p588. Decide whether the table of values

$x\backslash y$	0	1	2
0	0	5	10
1	2	7	12
2	4	9	14

could represent values for a linear function.

Solution. For each row, the slope $\dfrac{\Delta z}{\Delta y}$ is constantly 5. For each column, the slope $\dfrac{\Delta z}{\Delta x}$ is constantly 2. Thus the rows are linear with the same slope, and the columns are linear with the same slope. So, yes, these values could be those of a linear function.

Problem 5, §12.4, p588. The total charge, D, in dollars, to use an internet service is a function of m, the number of months of use, and t, the total number of minutes on line:

$$C = f(m,t) = 35 + 15m + 0.05t.$$

(a) Is f a linear function?
(b) Give units for the coefficients of m and t, and interpret them in terms of charges.
(c) Interpret the intercept 35 as a charge.
(d) Find $f(3, 800)$ and interpret your answer.

Solution. (a) Yes. It's of the form $f(m,t) = C + Am + Bt$, with A, B, C constant.
(b) We know that C is measured in dollars, m is measured in months, and t is measured in minutes. Therefore the coefficient, 15, attached to the m, carries the units dollars per month, while the coefficient, 0.05, attached to the t, carries the units dollars per second. The 15 dollars per month represents a monthly charge that is not affected by how many minutes the customer is on line. The 0.05 dollars per minute represents the per-minute cost for time actually spent on line.
(c) The 35 could be viewed as a sign-up cost for obtaining the online service. (It seems likely that the 35 dollars wouldn't be assessed except when the customer first signs up, but perhaps this is an unusual billing procedure. Or perhaps the 35 dollars represent an annual fee, for instance.)
(d) The formula for $f(m,t)$ gives $f(3, 800) = 35 + (15)(3) + (0.05)(800) = 120$. A customer who uses the service for 3 months and is on line for 800 minutes (or 13 hours and 20 minutes) during those three months would owe the company \$120 (or something under 10 dollars per hour).

Problem 7, §12.4, p588. Does the contour diagram shown in the text represent a linear function?

Solution. In this contour diagram the contours are all parallel lines. However, the contour lines are not equally spaced for equal increments of z. The space, for instance, between the line where $z = 0$ and the line where $z = 2$ is much smaller than the space between the line where $z = 2$ and the line where $z = 4$, but for linearity those gaps should be the same. The diagram cannot represent a linear function.

Problem 9, §12.4, p588. Does the contour diagram shown in the text represent a linear function?

Solution. The contour sets are all straight lines parallel to each other. Moreover, for equal increments of z the contour lines are equally spaced. So this diagram would be for a linear function.

Problem 11, §12.4, p588. (a) Find a formula for the linear function whose graph is a plane passing through the point (4,3,-2) with slope 5 in the x-direction and slope -3 in the y-direction. (b) Sketch a contour diagram for this function.

Solution. We can use the formula

$$z = f(x, y) = z_0 + m(x - x_0) + n(y - y_0)$$

with $(x_0, y_0, z_0) = (4, 3, -2), m = 5$, and $n = -3$. This gives

$$z = -2 + 5(x - 4) - 3(y - 3)$$

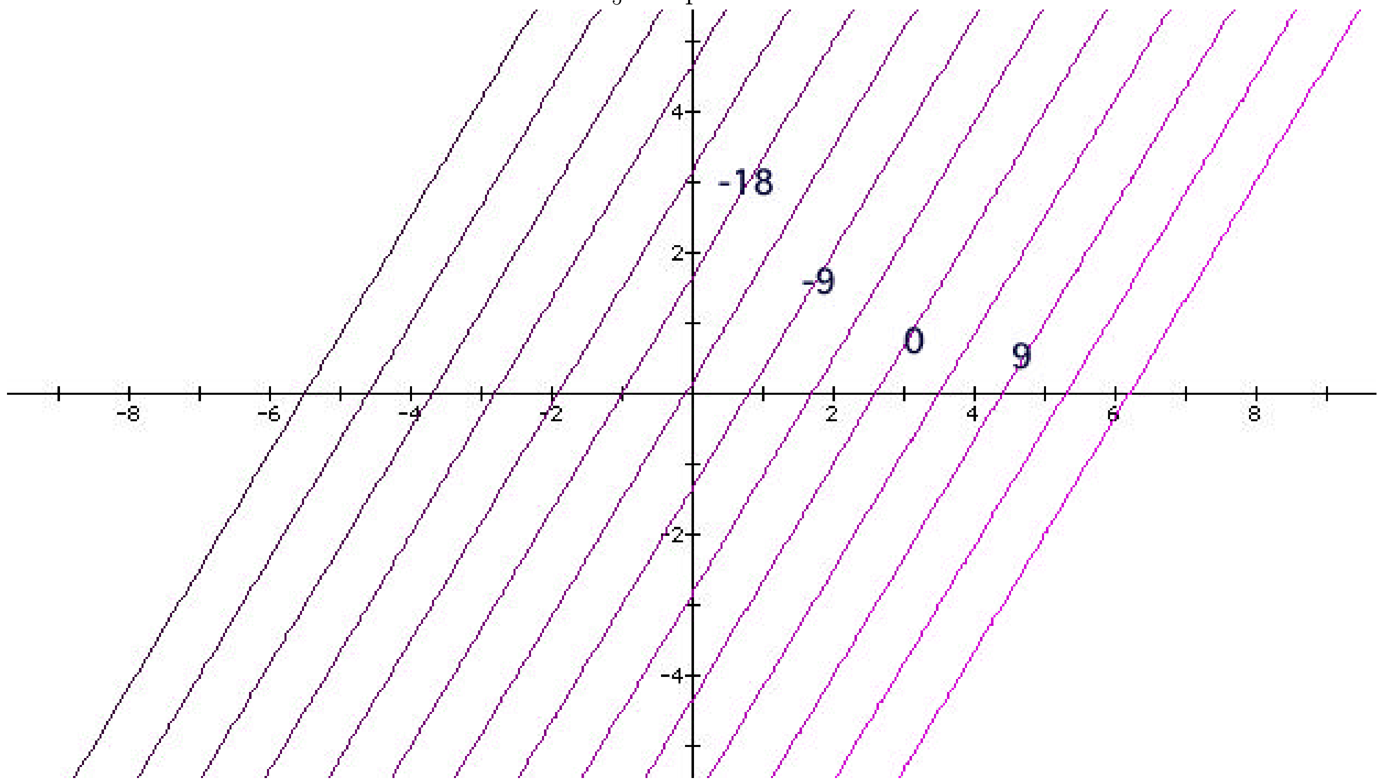
or

$$z = 5x - 3y - 13.$$

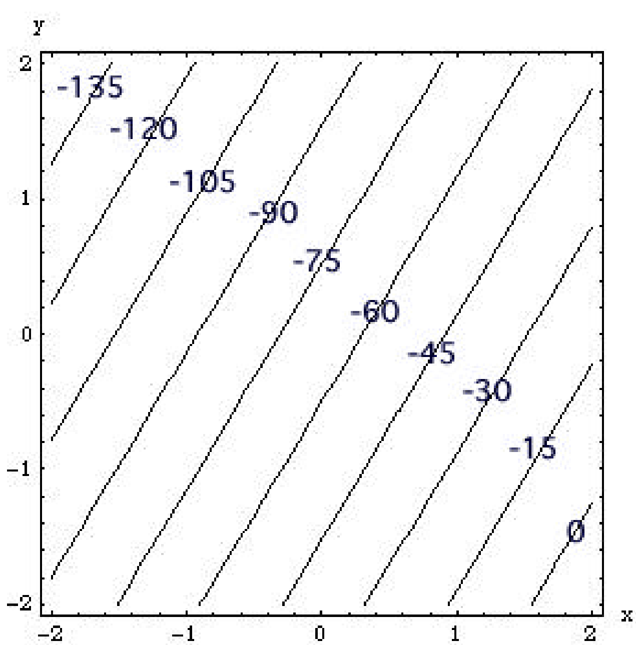
Fixing $z = c$ gives $5x - 3y - 13 = c$ or

$$y = \frac{5}{3}x - \frac{13}{3} - \frac{c}{3}.$$

These are straight lines with slope $\frac{5}{3}$. Furthermore, each time c goes up by 3, the y-intercept goes down by 1. Here's a possible contour diagram showing the contours for $c = 9, 0, -9$, and -18 (all labeled) plus several more values of c (not labeled).



Problem 13, §12.4, p588. Find an equation for a linear function with contour diagram



Solution. One way to proceed to find the slope in the x-direction, the slope in the y-direction, and a point on the corresponding plane. Line to line from left to right in the contour diagram, the z-values on the contour lines go up by 15. Line to line (and left to right), with x fixed, the y-values go down by 3 (for $x = 0$, look at the lines for z-levels -135 and -105 and see that y goes down by 6 as we move 2 lines over). Similarly, moving from line to line (left to right) with y fixed, the x-values go up by 3 (look at the lines for z-levels -75 and -15). So the slope in the y-direction is

$$n = \frac{15}{-3} = -5$$

while the slope in the x-direction is

$$m = \frac{15}{3} = 5.$$

Also, $f(4, 0) = -75$, so we can use

$$(x_0, y_0, z_0) = (4, 0, -75)$$

as a point on the desired plane. Using these values in the formula

$$z = f(x, y) = z_0 + m(x - x_0) + n(y - y_0)$$

we obtain a formula for our function:

$$z = f(x, y) = -75 + 5(x - 4) - 5(y - 0)$$

or

$$f(x, y) = -95 + 5x - 5y.$$

Problem 15, §12.4, p588. Find an equation for the linear function having the values shown in the table

$x\backslash y$	10	20	30	40
100	3	6	9	12
200	2	5	8	11
300	1	4	7	10
400	0	3	6	9

Solution. With y fixed, as x goes up by 100, z goes down by 1. So the slope in the x-direction is

$$m = -\frac{1}{100}.$$

Similarly, with x fixed, as y goes up by 10, z goes up by 3. So the slope in the y-direction is

$$n = \frac{3}{10}.$$

Using one of the entries in the table, see that, for instance, $f(400, 10) = 0$. Thus we can use

$$(x_0, y_0, z_0) = (400, 10, 0)$$

as a point on the graph of the function. Using these values in the formula

$$z = f(x, y) = z_0 + m(x - x_0) + n(y - y_0)$$

we find a formula for the function:

$$f(x, y) = -(0.01)(x - 400) + (0.3)(y - 10)$$

or

$$f(x, y) = -0.01x + 0.3y + 1.$$