Math 205
Quiz 2

1. (20 points) Let $\boldsymbol{c} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$, and let $\boldsymbol{d} = 2\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k}$.
   (a) Find the magnitude of $\boldsymbol{d}$.
   (b) Find the angle between $\boldsymbol{c}$ and $\boldsymbol{d}$.
   (c) Give two different vectors that are perpendicular to $\boldsymbol{c}$ and two different vectors that are parallel to $\boldsymbol{c}$.
   (d) Find the component of $\boldsymbol{c}$ that is parallel to $\boldsymbol{d}$.
   (e) Find the component of $\boldsymbol{c}$ that is perpendicular to $\boldsymbol{d}$.
   (f) Find an equation for the plane that passes through the point $(1, 1, 1)$ and has $\boldsymbol{c}$ as a normal vector.
2. (6 points) Show that $\lim_{(x,y)\to(0,0)} \dfrac{4xy}{3x^2 + 2y^2}$ does not exist.
   Hint: try exploring some limits along straight-line paths to $(0, 0)$.
3. (4 points) Let $f(x, y) = \dfrac{x^4 + x^2y^2 + y^4}{x^2 + y^2}$.
   (a) Briefly tell why
   $$0 \le f(x, y) \le \frac{x^4 + 2x^2y^2 + y^4}{x^2 + y^2}$$
   for $(x, y) \neq (0, 0)$.
   (b) Verify that
   $$\lim_{(x,y)\to(0,0)} \frac{x^4 + 2x^2y^2 + y^4}{x^2 + y^2} = 0.$$
   Hint: first use a little algebra to rewrite the desired limit.
   (c) Based on the information we have now collected, what would you say (and why) is the value for $\lim_{(x,y)\to(0,0)} f(x, y)$?