Math 205
Quiz 3 with Answers

1. (6 points) Find an equation for the plane through the points $(1, 0, -2), (2, 1, 1), (1, 4, 3)$.

2. (4 points) Evaluate the determinant for the 4×4 matrix

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

3. (6 points) Find an equation for the plane tangent to the surface $z = \sin(xy) + 4x$ at the point where $x = 1$ and $y = \pi$.

4. (9 points)
   (a) Find the second-order Taylor polynomial $Q(x, y)$ for the function $f(x, y) = \cos 2x + xy$ using the base point (0,1).
   (b) Use this Taylor polynomial to find an approximate numerical value for $f(0.1,\ 0.9)$.

5. (15 points) Let $F(x, y, z) = \mathrm{e}^{x^2+y^2-z}$.
   (a) Find the gradient vector for F at the point $(1, 1, 2)$.
   (b) Find the directional derivative for F at the point (1,1,2) in the direction given by the vector $\overrightarrow{i} + \overrightarrow{j} + \overrightarrow{k}$.
   (c) At the point $(1, 1, 2)$, in which direction does F decrease most rapidly?

Answers.
1. $7x + 5y - 4z = 15$
2. -1
3. $z = 4 + (4 - \pi)(x - 1) - 1(y - \pi)$ or $z = (4 - \pi)x - y + 2\pi$
4(a). $Q(x, y) = 1 + x - 2x^2 + x(y - 1)$ or $Q(x, y) = 1 - 2x^2 + xy$
4(b) 1.07
5(a). $2\overrightarrow{i} + 2\overrightarrow{j} - \overrightarrow{k}$
5(b) $\sqrt{3}$
5(c) In the direction opposite to the gradient, hence in the direction given by the vector $-2\overrightarrow{i} - 2\overrightarrow{j} + \overrightarrow{k}$.

 A. Sontag March 7, 2004