Math 200
Some Notation and Terminology
Sets and Sets of Numbers

Most of mathematics is formulated in the language of set theory. A careful treatment of set theory begins with "set" and "is an element of" or "belongs to" as undefined terms and posits certain axioms that describe what we assume about sets and elements of sets. We will adopt a more informal attitude, essentially assuming that we know what the word "set" means: a set is a collection of distinct objects, which we call the elements or members of the set. (Notice the circularity here!) It is essential to this understanding that the membership criterion for belonging to a set must be well determined. An object cannot "sort of be a member of" a set, or partly belong to a set, or sometimes belong and sometime not. It either is a member of the set or it isn't, though we may not know which.

$\in$. If S is a set and x is an element of S (in different words, x belongs to S or x is a member of S), we indicate this by writing $x \in S$.

$=$. Two sets S and T are said to be equal, written $S = T$, if and only if they have exactly the same elements or members. In other words $S = T$ means that each element of S also belongs to T and vice versa. For instance, the set S whose members are the current U.S. Senators from the state of Massachusetts is equal to the set T whose members are Edward Kennedy and John Kerry. This example also illustrates that one and the same set may be defined in lots of different ways.

$\emptyset$. The empty set is the set with no members at all. We shall denote the empty set by $\emptyset$.

$\subseteq$. For S and T sets, if every element of S is also an element of T, then we say that S is a subset of T, and we write $S \subseteq T$. Thus, $S = T$ is equivalent to $S \subseteq T$ and $T \subseteq S$. Notice that, for every set S, $\emptyset \subseteq S$ and $S \subseteq S$.

If $S \subseteq T$, we sometimes also say that T contains S, but this should not be confused with $S \in T$, which is also occasionally described by saying that T contains S. It is probably better to say, instead, T contains S as a subset (to indicate $S \subseteq T$) or T has S as an element (to indicate $S \in T$).

$\subset$. For S and T sets, we say that S is a proper subset of T if and only if $S \subseteq T$ but $S \neq T$. If S is a proper subset of T, we write $S \subset T$. Thus $S \subset T$ is equivalent to $S \subseteq T$ and $S \neq T$. For instance, if S denotes the set whose members are the current U.S. Senators from the state of Massachusetts, and T denotes the set whose members are the current U.S. Senators, then $S \subset T$ (in other words, S is a proper subset of T). The empty set is the only set having no proper subsets.

$\nsubseteq$ and $\not\subset$. We'll also write $S \nsubseteq T$ to indicate that S is not a subset of T. Similarly, we write $S \not\subset T$ to indicate that S is not a proper subset of T (in which case S might or might not be a subset of T).

$\cap$. For S and T sets, we write $S \cap T$ to denote the set whose members are those objects that belong to both S and T. We read $S \cap T$ as "the intersection of S and T" (or sometimes simply S intersected with T, or even S intersect T). If $S \cap T = \emptyset$, we say that S and T are disjoint. In other words, two sets are disjoint if they have no members in common.

$\cup$. For S and T sets, we write $S \cup T$ to denote the set whose members are those objects that belong to S or T (or both). $S \cup T$ is called the union of S and T.

$\backslash$. For S and T sets, we write $S \backslash T$ to denote the set whose members are those objects that belong to S but not to T. Some like to read $S \backslash T$ as "S, not T". A better, if more cumbersome, name for it is "the set-theoretic difference S minus T" which we shorten to "S minus T" when there's no room for confusion. We might also read it as "S backslash T". As an example, if S is the set whose members are the current U.S. Senators from New England, and T is the set whose members are the current U.S. Senators from the states of Massachusetts, California, Colorado, and Minnesota, then $S \backslash T$ is the set whose members are the 10 current U.S. Senators from the other five New England states (Connecticut, Rhode Island, Vermont, New Hampshire, and Maine).

Complement. For S and T sets, if $T \subset S$, then the set $S \backslash T$ is also called the complement of T relative to S. In a context where all sets under consideration are subsets of some "universal set" U, we simply refer to $U \backslash S$ as the complement of S.

Power Set. For S a set, we denote by $\wp(S)$ will denote the set whose members are the subsets of S, and we call $\wp(S)$ the power set of S. In other words, $A \in \wp(s)$ means the same as $A \subseteq S$. In particular, for every set S we have $\emptyset \in \wp(S)$ and $S \in \wp(S)$.

You are already familiar with several important sets of numbers, namely the set of natural numbers (or counting numbers), the set of integers (or whole numbers), the set of rational numbers, the set of real numbers, and the set of complex numbers (sometimes called imaginary numbers). We shall denote these sets by

N (think: N for natural)
Z (Z for Zahlen, which is German for numbers, positive or negative)
Q (Q for quotient or ratio, so rational)
R (R for real), and
C (C for complex).

Note that the words "natural, rational, real, complex" don't necessarily have the same meaning here as they do in everyday English, although they were originally chosen because of some everyday connotation.

You are probably familiar with such notation as the following:

$S = \{1, 2, 3\}$ specifies that S is the set whose members are the numbers 1, 2, and 3
$S = \{0, 1, 2, 3, ...\}$ specifies that S is the set whose members are the counting numbers; in other words $\{0, 1, 2, 3, ...\}$ is the same set as **N**.
$S = \{x \in \mathbf{Z} | x \geq 0\}$ or $S = \{x \in \mathbf{Z} | x \geq 0\}$ specifies that S is the set whose members are those integers that are greater than or equal to 0; in other words $\mathbf{N} = \{x \in \mathbf{Z} | x \geq 0\} = \{x \in \mathbf{Z} | x \geq 0\}$.

Interval Notation. For a, b real numbers with $a < b$, $[a, b] := \{x \in \mathbf{R} | a \leq x \leq b\}$ and $(a, b) := \{x \in \mathbf{R} | a < x < b\}$. For $a \in \mathbf{R}$, $(a\, \infty) := \{x \in \mathbf{R} | a < x\}$ and $(-\infty, a) := \{x \in \mathbf{R} | x < a\}$. Similarly, $[a, \infty) := \{x \in \mathbf{R} | a \leq x\}$ and $(-\infty, a] := \{x \in \mathbf{R} | x \leq a\}$.