Pitch center of stringed instrument vibrato tones

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The determination of the pitch center of frequency-modulated sounds has been the focus of a number of previous studies. The sources have usually been pure tones or synthetic complex sounds with a well-defined spectral composition. These synthetic sounds differ in temporal and spectral properties from the sounds produced by musical instruments; and it is these acoustic sounds which performers are trained to produce and to perceive in order to make intonation choices. Thus samples chosen for this study consist of approximately 1 s of acoustic sounds produced by a virtuoso violist playing the notes D_4, $C_5\sharp$, A_5, and G_6 with and without vibrato. The sounds without vibrato were then resampled to give frequencies from -15 to $+21$ cents with respect to the mean of the sound with vibrato. Two-interval two-alternative forced choice (2I2AFC) experiments were carried out comparing the sounds with vibrato to those without vibrato using two sets of musically experienced listeners as subjects. A control set consisting of the comparison of pitch levels of the unmodulated sounds was carried out simultaneously. Results are consistent with the finding that the pitch perceived is that of the mean. The difference limen inferred from the control set was 2.8 cents for the first group and 2.5 cents for the second group with an upper bound on the error of 1 cent.

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BACKGROUND

The problem of determining the pitch center or the perceived pitch of frequency modulated sounds has been studied over a long period of time by a number of workers. The problem is of interest to psychoacousticians for insight into the mechanisms of pitch perception. An understanding is also necessary for the study of intonation choices by string performers since most of their notes are played with vibrato. In fact for a meaningful study of intonation, the following questions must be answered:

> What pitch is perceived by experienced musical performers and listeners when a musical sound with vibrato is presented?
>
> How do the accuracy and standard deviation of the responses of these experts compare for modulated and unmodulated sounds?

More fundamentally, in order to even pose the above questions or to study any question related to perception of musical sounds, it is essential to know:

> How constant a frequency is it possible to produce with a musical instrument?

In this study we will address these questions with an emphasis on the first two. As a partial answer to the last question we report results on one performer.

The results of previous studies of pitch perception of frequency modulated sounds are presented in inverse chronological order in Table I. The three most recent studies (Iwamiya *et al.*, 1983; Shonle and Horan, 1980; and Sunberg, 1978) in Table I appear to agree that the mean pitch is perceived although there is some discussion as to whether it is the geometric or arithmetic mean. All three experiments were conducted using the method of adjustment which has certain problems in its rational underpinnings (Hake and Rodwan, 1966).

Little attention has been paid to the use of musically trained subjects known to have ''good ears.'' One study uses a group of ''novices'' which were undergraduate students picked at random and so-called ''experts'' who were graduate students in music, physics, mathematics, or engineering; the tacit assumption being that scientific talent translates into expertise in musical perception. Insufficient emphasis has been paid to the inherent accuracy of the experiments. All were conducted using synthesized sounds. Shonle and Horan (1980) include an excellent review of early work in this area.

In some of these studies (entries 4, 5, 6 in Table I) sounds were produced with and without vibrato by a performer, and these were considered to be his or her choice of the same pitch. We include in Sec. II a similar study, which is an extensive analysis of sounds produced by a well-known concert violist (to be referred to as MT).

In more recent studies, Iwamiya *et al.* (1985, 1989) have studied the effect of simultaneous amplitude modulation and frequency modulation of the components of two and three component complexes. They find that there is a pitch shift which depends on the relative phases of the AM and FM components. These results are not included in Table I since this is a different experiment from those reviewed, but will

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TABLE I. Summary of previous work. The conclusion column indicates the position of the pitch center of the vibrato. Relative terms denote pitch height with respect to the mean of the vibrato. Abbreviations: s—sharp, f—flat, LPF—low-pass filtered.

	Fund frequency (Hz)	Method	Sound	Conclusion
Iwamiya *et al.* (1983)	440–1500	adjustment	sinusoid	mean
Shonle and Horan (1980)	440;1500	adjustment	sin; known complex	mean
Sundberg (1978)	300–770	adjustment	synthetic baritone	mean
Beauchamp (1974)				1 sharp; 1 mean
Fletcher and Sanders (1967)				flat
Fletcher *et al.* (1965)				sharp
Seashore (1938)	240;480	const stimuli	air siren	ave of extremes
Hirose (1934)	900–1500	const stimuli	nonsinusoidal	s or f dep on vib extent
Tiffin (1931)	420	const stimuli	periodic LPF pulses	flat

be discussed later in Sec. III. See also Anantharaman *et al.* (1993).

d'Alessandro and Castellengo (1994) have studied the perception of very short time segments of synthesized vocal vibrato. They conclude that the pitch perceived depends on the final phase of the vibrato, and propose a weighted time average theory where the end of the tone is weighted.

In his 1970 review of musical perception, Ward states ''Studies matching the pitch of frequency-modulated sinusoids (or of actual musical tones) to that of steady tones are badly needed.'' Two and one half decades later there have been no studies using musical sounds produced by real instruments although it is on these sounds that performers have spent vast amounts of time training as they tune their instruments. They are thus the most experienced subjects ever to enter a booth as well as subjects with a known proficiency in listening tasks.

It was also felt that a study with a musical emphasis is a more meaningful way to address the question of the jnd of natural sounds since it is on these sounds that experts are trained. The current study differs from the previous ones in that it is conducted with actual musical sounds. The subjects all have experience performing with musical instruments where tuning is a necessary part of performance (in contrast to playing keyboard instruments where the performer is not responsible for the intonation). The experimental method is that of two-interval two-alternative forced choice which has advantages over the method of adjustment. There are no calibration problems or questions of motor control in adjusting a dial or a mouse. And even if carried out perfectly, with the method of adjustment there is a question of whether some subjects were more tenacious in seeking exactly the right match. In 2I2AFC there is a clean choice made by the subject between two known sounds as to which has the higher pitch.

In reporting the results, the performers are divided into two groups according to their musical experience. The first group are nonprofessional performers from the MIT Media Lab; whereas the second group consists of graduate students studying violin at New England Conservatory (NEC) and a professional violinist from the Boston area.

Preliminary results in this study were reported by Brown and Vaughn (1993).

I. EXPERIMENTAL METHOD

A. Sound production and manipulation

All of the sounds used in this study were recorded digitally at a 44 100 sampling rate in our studio with the virtuoso violist (MT) playing a number of notes both with and without vibrato on the viola. They were analyzed using the high-resolution fundamental frequency tracker of Brown and Puckette (1993). Sound segments were chosen which represented a range of frequencies and which had a standard deviation in frequency of 2 cents or less. They are included in Table II with their frequencies in column 3. The notes A_5 and $C_5\sharp$ were originally recorded as G_5 and $E_5\flat$ and resampled to give a more uniform spacing of the four frequencies chosen for the experiment. The resampling software is part of Dan Ellis's dspB software. It consists of a Hanning-windowed ideal sinc interpolator in a polyphase rational-intermediate-frequency implementation, where both the window length and the sampling-frequency ratio are chosen to allow arbitrary accuracy in aliasing rejection and output sampling rate. These two resampled sounds had a vibrato rate which was altered by about 12%, but they sounded natural

TABLE II. Properties of sounds used in the study. The frequency is given in Hz with the standard deviation and modulation amplitude in cents. Columns 6–16 give the intensity in dB for harmonics with number corresponding to the labeling of the column. Abbreviations: s.d.—standard deviation, unmod—unmodulated sound, mod—frequency modulated sound, amp—amplitude.

Note	Midinote	Frequency (Hz)	s.d. unmod (cents)	Mod amp (cents)	1	2	3	4	5	6	7	8	9	10	11
D_4	62	293.3	0.764	15	0	0	−8	−37	−8	−8	−8	−10	−32	−18	−14
C_5 #	73	551.8	1.321	25	−5	−18	−10	0	−22	−13	−20				
A_5	81	878.5	0.823	27	0	−10	−8	−16	−13						
G_6	91	1580.7	2.042	15	0	−17	−10								

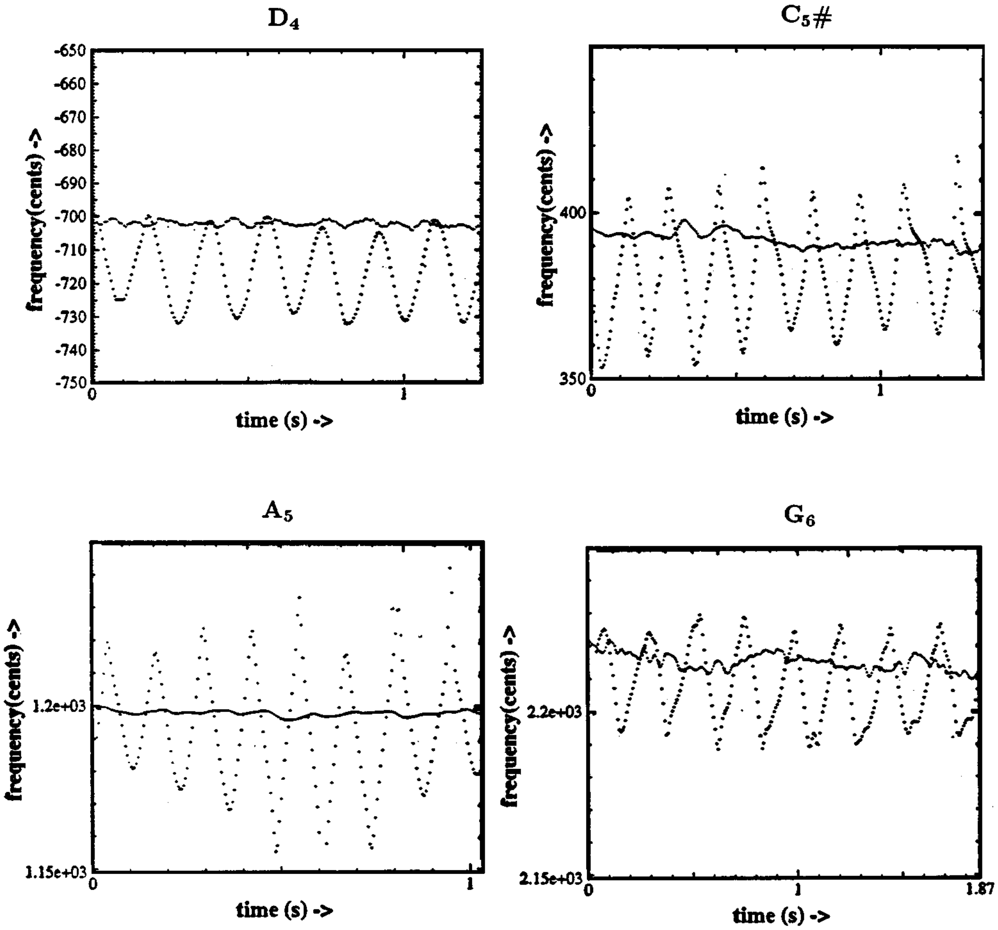

FIG. 1. Frequency versus time for the unmodulated notes D_4, $C_5\sharp$, A_5, and G_6 with the results for the modulated notes superposed on them.

and were not commented on by the performer nor any of the other subjects in the listening test. The rates were approximately D_4 5.5 Hz, $C_5\sharp$ 6.0 Hz, A_5 7.7 Hz, and G_6 4.4 Hz. Graphs of frequency versus time for all of these sound segments are found in Fig. 1 with the results for the unmodulated note superposed on the modulated one. Frequencies are in cents with respect to A440.

Average intensity levels in dB for the unmodulated sounds are reported in columns 6–16 of Table II, where the number labeling the column is the harmonic number. Intensities are given relative to that of the most intense component, which is assigned the value 0 dB. The values of components are given up to a harmonic number beyond which all intensities are down by 20 dB or more. The (maximum) amplitudes of the component labeled 0 dB (16 bit sample values out of a possible 32 768) are D_4 9000, $C_5\sharp$ 12 000, A_5 13 000, and G_6 9500. For example, for the note D_4 in Table II, the first and second harmonics have amplitudes of 9000, and any of the other amplitudes could be calculated from the dB values in the table.

The geometric means of the vibrato sounds were calculated from 172 values/s. These values were calculated from overlapping 50-ms segments of sound. For the vibrato amplitudes of this study the geometric and arithmetic means are equivalent (differences on the order of 0.1 cent) and will be referred to simply as the mean. The vibrato was then labeled by this mean frequency for purposes of identification. The unmodulated sound was resampled to have the mean frequency and frequencies equal to the mean ± 3, ± 6, ± 9, ± 15, and $+21$ cents. The positive asymmetry was chosen as some string players claim to hear the high side of the vibrato. With this system of nomenclature then, a match of the vibrato pitch to the unmodulated sound labeled $+15$ would mean that the pitch center of the vibrato is 15 cents above its mean for that subject.

B. Listening experiment

Each listener was presented with the notes D_4, $C_5\sharp$, A_5, and G_6 with or without vibrato followed by the same note without vibrato at ten pitch levels in a randomized order. As mentioned earlier the mean of the vibrato note corresponds to the 0 level of the nonvibrato note. There were eight repetitions of each comparison for a total of 640 trials for each person. The sound level was adjusted to a comfortable level by the subject. The stimuli were presented for approximately 1 s with an interstimulus interval of 250 ms. They were played through a Digidesign Audiomedia I board in mono using AKG k-240 monitor headphones. After hearing the two sounds, the subject clicked the appropriate box on the screen. Choices were definitely higher, maybe higher, maybe lower, or definitely lower. The subject could then click a box for the continuation of the experiment. The responses were merged to simply higher or lower for data analysis and graphs.

TABLE III. Summary of frequency tracking results on viola sounds. All frequencies are given in cents with respect to A 440. Abbreviations: ETD—difference from equal temperament, Vnv diff—vibrato–nonvibrato, Vib PP—peak-to-peak amplitude of the vibrato.

Note	Freq (cents)	ETD	Vnv Diff	Vib PP	Note	Freq (cents)	ET D	Vnv Diff	Vib PP
A_4 open	−2				D_4 open	−702			
A_4 stpd	8	8			D_4 stpd	−705	−5		
A_4 vib	3	3	−5		D_4 vib	−716	−16	−11	30
A_4 stpd (2)	5.5				D_4 stpd (2)	−706			
G_3	−1400	0			G_3 (2)	−1414	−14		
G_3 vib	−1414	−14	−14	34	G_3 vib (2)	−1422	−22	−8	38
G_4	−200	0			G_4 (2)	−208	−8		
G_4 vib	−200	0	0		G_4 vib (2)	−208	−8	0	
G_5	998	−2			G_5 (2)	998	−2		
G_5 vib	998	−2	0	76	G_5 vib (2)	988	−12	−10	36
G_6	2200	0			G_6 (2)	2213	13		
G_6 vib	2208	8	8	37	G_6 vib (2)	2214	14	1	43
$E_3\flat$	−1816	−16			$E_3\flat$ (2)	−1815	−15		
$E_3\flat$ vib	−1836	−36	−20	43	$E_3\flat$ vib (2)	−1827	−27	−12	33
$E_4\flat$	−618	−18			$E_4\flat$ (2)	−614	−14		
$E_4\flat$ vib	−617	−17	1	100	$E_4\flat$ vib (2)	−611	−11	3	80
$E_5\flat$	592	−8			$E_5\flat$ (2)	593	−7		
$E_5\flat$ vib	588	−12	−4	65	$E_5\flat$ vib (2)	583	−17	−10	57
$E_6\flat$	1783	−17			$E_6\flat$ (2)	1790	−10		
$E_6\flat$ vib	1794	−6	11	40	$E_6\flat$ vib (2)	1798	−2	8	40

II. RESULTS

A. Experiment 1: Intention of the performer

1. Description of the experiment

The violist was asked to verify his tuning with an A440 standard sound and then to play his A string as an open string, then to play the same frequency stopped on a second string, then with vibrato, and then stopped a second time. He then played a series of G's (starting with G_3) first without and then with vibrato. This series was repeated. The same procedure was followed for E♭ played in octaves without and with vibrato. Finally the original experiment on A440 was played on the D string.

We did not obtain the extensive repetitions which would be necessary for statistically significant data on each note due to time constraints of the performer, and the fact that our principal object was to obtain a variety of pitches for use in our listening experiment. We nevertheless find these data interesting and are including it as Experiment 1; it is particularly useful in conjunction with Experiment 4, which reports a listening test on this performer.

A summary of all the pitch tracking results on the viola sounds are found in Table III. They were read from graphs of frequency in cents (with respect to A440) versus time such as those of Fig. 1. Uncertainties are less than 2 cents. They are entered with one exception in the order in which they were played. Repetitions of the same note are marked ''(2)'' in Table III and entered so that the same notes are on the same line of the table and their frequencies can be easily compared.

For convenience the D's are entered in the table in a column parallel to the results on the A's. This is an identical experiment although on a different note.

2. Qualitative observations

The frequency differences of notes played with and without vibrato are on the order of 10 cents and occur with both signs. If there is a systematic trend, it is not apparent. It is nevertheless interesting to note that the difference has the same sign for all pairs of the same note (if we discount 0 and −10).

We calculated the frequencies of all the notes with respect to their equal tempered values (column 3 of Table III), and ran an ANOVA on these data with note and vibrato/nonvibrato (vib/nv) as variables. Significance at the $p<0.001$ level was obtained for note, but no significance ($p=0.121$) for vib/nv as anticipated from the data. This means that the frequency differences from equal temperament, or intonation of the individual notes, are more closely correlated than are the members of the sets of notes played with and those played without vibrato; and there is no significant difference in notes played with and notes played without vibrato.

3. Conclusion

The overall average of deviations from equal temperament (column 3 of Table III) is −8.1 cents with a standard deviation of 10.7. The overall average of notes played without vibrato is -6 ± 9 cents and with vibrato -10 ± 12. So there is an upper bound of about 10 cents in the frequency control of the performer. Since the average vibrato peak to peak amplitude is 40 cents (see Table II), this difference is sufficiently large that if the violist had aimed for the highest or lowest points of the frequency modulation, this would have shown up in the data. We can thus conclude that our results are consistent with the intended pitch of the performer being the mean of the vibrato.

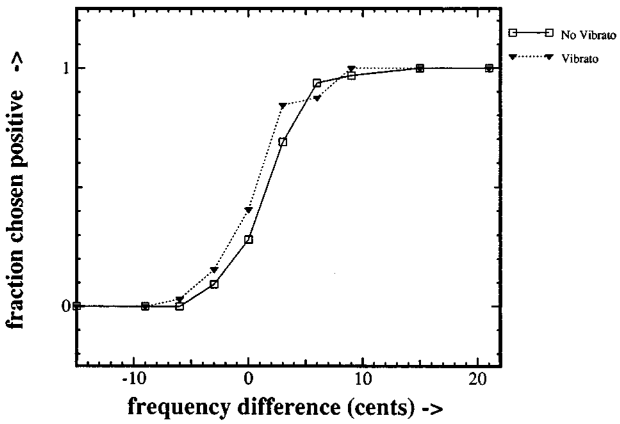

FIG. 2. Fraction of responses higher plotted against target pitch where 0 corresponds to the mean of the vibrato for the most accurate MIT subject.

B. Experiment 2: Nonprofessional performers

The first group of listeners consisted of six musicians from the MIT Media Lab. None were professional performers, but all were experienced in playing an instrument where intonation choices are made. Four of the six listeners were in their 20s. None had absolute pitch.

The psychometric curve for the most accurate listener in this group is presented as an example in Fig. 2, where we have plotted fraction of positive responses vs pitch level of the second sound (target pitch). For this and all other curves uncertain responses were given the same weight as certain responses. Thus if the mean of second sound is in fact at a higher frequency (positive half of the abscissa), then eight responses of higher would correspond to an ordinate of 1. Similarly for the mean of the second sound at a lower frequency, a score of all replies lower would correspond to an ordinate 0.

Although the other individual psychometric curves are not presented, we have summarized the individual data in Table IV. For this table the abscissa corresponding to an ordinate of 0.5 (meaning 50% responses higher) is read from each of the individual curves. This represents the pitch level which is judged to be the same as that of the vibrato sound. Recall that with our notation the vibrato sound is labeled by its mean so if an ordinate of 0.5 occurs at a value of 0 on the abscissa, this signifies that the pitch center of the vibrato is its mean for that listener. For the nonvibrato curve, identical sounds are being compared for the pitch level 0 position and any deviation from ordinate 0.5 is statistical or indicates bias on the part of the listener. This will be discussed in a later section.

The data in Table IV show that all listeners have a 50% point for the vibrato within 2 cents of their nonvibrato with the magnitude of the average difference between vibrato and nonvibrato 0.5 cents. It should thus be clear that the combined psychometric curve does not simply average out different modes of perception by different subjects. One of the subjects (S2) took the listening test twice. The second test is not averaged into the data but is included in Table IV for comparison. The second set of results were very consistent with the first set.

TABLE IV. Pitch level corresponding to 50% chosen higher. Columns 1 and 2 are the pitch level in cents for vibrato and nonvibrato, and column 3 is their difference. Subjects are labeled by experiment; e.g., S1expt2 is the data from subject 1 in Experiment 2.

Subject	Vib	NV	Vib−nv
S1 expt2	1.5	0.5	1
S2expt2	−1	1	−2
S3expt2	0	0.5	−0.5
S4expt2	−1	1	−2
S5expt2	2	3	−1
S6expt2	0	1	−1
S2expt2	−0.5	−1	0.5
TOTexpt2	0.5	1	−0.5
S1expt3	1	−1	2
S2expt3	4	4	0
S3expt3	1.5	2.5	−1
S4expt3	0	−0.5	0.5
S5expt3	−0.5	1.5	−2
TOTexpt3	0.5	0.5	0
Performer	0.5	2.5	−1.5

The average psychometric curve for this group is found in Fig. 3. The similarity of the vibrato and nonvibrato curves is very striking; the two curves are effectively identical. This is all the more impressive in view of the fact that the peak to peak vibrato amplitude ranges up to twice the total extent of the *x* axis. The average peak to peak amplitude of the vibrato is indicated on the curve. Yet the similarity of the curves would imply that the *average* frequency of this vibrato sound is perceived in exactly the same manner as the single frequency sound. There is not even a difference in the shape of the curve, which would have indicated more uncertainty in identifying the pitch of the vibrato.

C. Experiment 3 : Graduate level and professional violinists

A listening experiment identical to that described in the previous section was performed with four graduate students

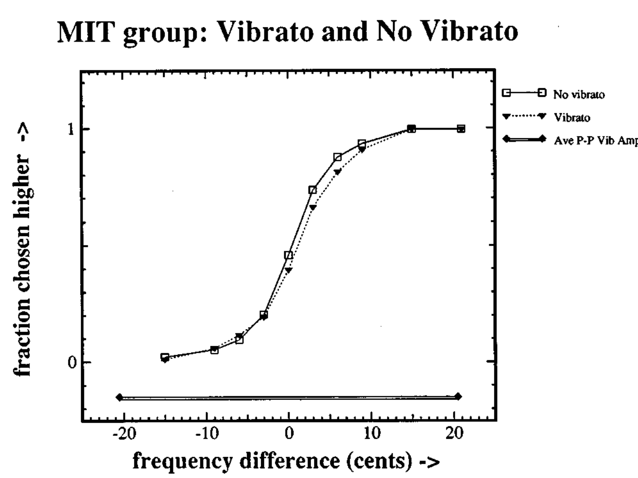

FIG. 3. Fraction of responses higher plotted against target pitch where 0 corresponds to the mean of the vibrato for the subjects of experiment 2. The average peak to peak amplitude is included below the curve for comparison.

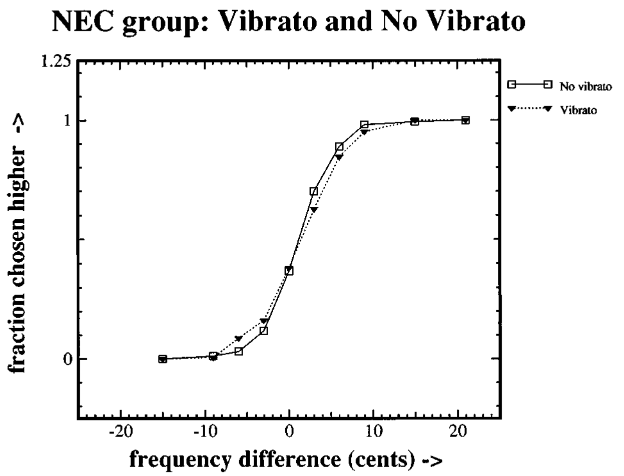

FIG. 4. Fraction of responses higher plotted against target pitch where 0 corresponds to the mean of the vibrato for string players. See Fig. 3 for a comparison to the average peak to peak amplitude of the vibrato.

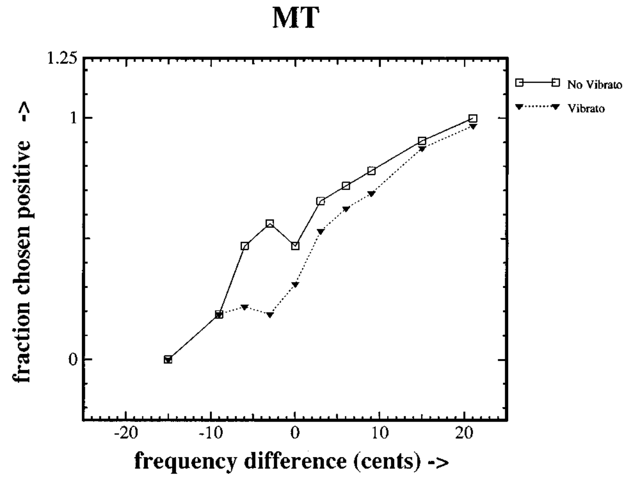

FIG. 5. Fraction of responses higher plotted against target pitch where 0 corresponds to the mean of the vibrato for the performer discussed in Experiment 1 and Experiment 4.

studying violin at New England Conservatory (NEC) and a professional violinist from the Boston area, the goal being to determine whether string players perceive vibrato produced by stringed instruments in the same way that other musicians do. One of the graduate students professed to have absolute pitch, but his results did not differ from the others, so they are included. The results on the individual subjects are found in Table IV with their average psychometric curve in Fig. 4. As was found for the first group of listeners, the data support the conclusion that the pitch center of the vibrato is at its mean. The psychometric curve is steeper around pitch level 0 indicating that these subjects are a little better at pitch discrimination than the first group. Alternatively this could be due to the fact that the sounds were produced by a stringed instrument, and these listeners had had far more experience judging intonation of these particular sounds.

D. Experiment 4: Listening test of the performer

We were fortunate in being able to run our listening test on the performer who generated the sounds discussed in Experiment 1, and we are very grateful to him for his participation. The results on his 50% points are in Table IV with his psychometric curve in Fig. 5. These data are slightly less accurate than those of other listeners, and are consistent with the 10 cent uncertainty discussed in the conclusion of Experiment 1.

III. GENERAL COMMENTS AND CONCLUSIONS

One question which arises in this study is whether the simultaneous amplitude modulation, which necessarily accompanies frequency modulation when vibrato is played on a violin, has an effect on our results. Previous results on synthetic sounds (Iwamiya and Fujiwara, 1985) have indicated that the relative phases of the AM and FM do affect the pitch perceived. If this is true for a given note, we would expect to find the average of the judgments for vibrato differing from the average for nonvibrato by more than a standard deviation.

In Fig. 6 we have plotted the average value of fraction chosen positive for vibrato and nonvibrato for each of the four notes of the study against midinote. The midinote is calculated by adding the number of semitones a note lies above middle C to the number 60 assigned to middle C. We should note that for the comparison of identical sounds for the perfect listener (all correct responses and 50% higher for the case of the same pitch levels), the average would be 0.55 reflecting 5 positive levels and one match. The standard deviation for each of the points is 0.4 and only the note A_5 played with vibrato lies more than 0.5 below this ''correct'' value meaning it is perceived higher.

It is tempting to compare the phase of the overall amplitude waveform to that of the frequency variation of Fig. 1 to determine whether the high intensity coincides with the high-frequency side of the vibrato or vice versa. However, vibrato sounds produced by stringed instruments are extremely complex, with the intensities of the different frequency components varying independently (due to the for-

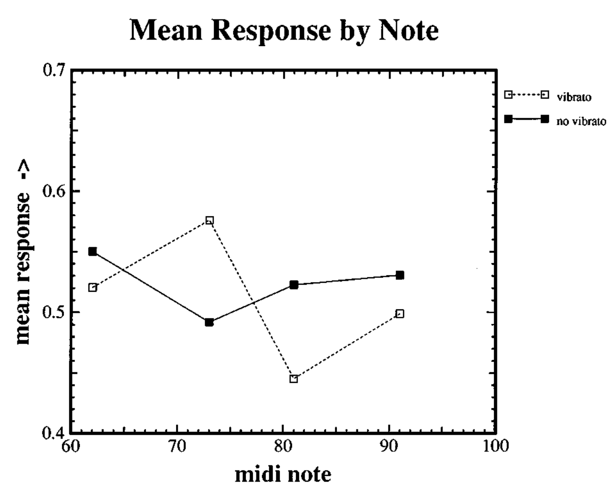

FIG. 6. Average of fraction higher for all responses by pitch class and vibrato nonvibrato plotted against note given as midinote. (Notes are D_4, $C_5\sharp$, A_5, and G_6.) For the perfect listener this average would be 0.55.

TABLE V. Responses for comparison of same sound. Abbreviations: DL—definitely lower, ML—maybe lower, MH—maybe higher, DH—definitely higher.

Subject	DL	ML	MH	DH
S1expt2	0	23	9	0
S2expt2	0	14	13	5
S3expt2	0	17	14	1
S4expt2	1	10	21	0
S5expt2	0	22	10	0
S6expt2	2	15	15	0
TOTexpt2	3	101	82	6
S1expt3	4	20	6	2
S2expt3	18	7	4	3
S3expt3	6	15	10	1
S4expt3	7	10	10	5
S5expt3	5	9	12	6
TOTexpt3	40	61	42	17
Performer	2	15	12	3

mants of the body of the instrument) rather than in phase with each other (Mathews and Kohut, 1973; Jansson, 1980; Brown, 1996). This is the source of the very rich sound produced by stringed instruments.

We have examined the intensity levels of the harmonic components of A5 since the effect is largest for this note, and find that the fundamental is 10 dB over the other harmonics and is out of phase with the high-frequency side of the vibrato. This would have the effect of lowering the perceived pitch of the vibrato according to Iwamiya and Fujiwara (1985) whereas the results of Fig. 6 indicate that it is perceived somewhat higher. This is a small effect on a sound with a complicated harmonic structure. More careful and controlled experiments are called for in order to explore this effect.

A computer program was written to keep track of the responses for the comparison of identical sounds. There were 32 such pairs for each person, and ideally there would be roughly half ''maybe higher'' and half ''maybe lower'' responses. A heavier weighting toward higher or lower responses would indicate a bias toward one choice in the results. We include these data in Table V.

It is interesting that the string players have a high number of ''definitely higher'' and ''definitely lower'' responses for these identical sounds. Since the other results from this group were slightly better than those of the first group, this is puzzling. Their earlier results show clearly that they perceive extremely small differences accurately, but they seem unable to recognize (or possibly to admit to) the situations in which certainty is not possible.

Although the principal goal of this study was the determination of the pitch center of musical notes played with vibrato, the simultaneous control experiment comparing unmodulated sounds provides an estimate of the jnd for frequency for these subjects. From signal detection theory the 76% correct point on the psychometric curve corresponds to $d'=1$ (Moore, 1989; Durlach, 1968) for a 2I2AFC experiment without response bias. The frequency separation at this point is an estimate of the jnd. With our scale from 0 to 1 these would be the frequency differences on the abscissa corresponding to points 0.24 and 0.76 on the psychometric curves. Averaging over the values at these two points, we found 2.8 cents for the MIT group and 2.5 cents for the NEC group with an upper bound on the error of ± 1 cent. The error is estimated from differences in the values at 0.24 and 0.76 which represent the same sounds heard in reverse order.

These values are slightly lower than previous values of 3.5–4 cents for pure tones summarized by Moore (1989), as would be expected for complex sounds (Spiegel and Watson, 1984). They fall within the range 1.7 to 7.5 cents reported by Spiegel and Watson (1984) for musicians discriminating square wave stimuli though they are smaller than his average values of 4.5 and 5.0 cents for frequencies 430–910 Hz. Moore and Glasberg (1990) report a DL of roughly 3 cents for complex tones containing the first six harmonics.

Although our results are in agreement with previous studies, it should be recalled that our experiments involve stimuli with a small frequency variation inherent in the use of actual musical sounds. In fact, our jnd's are only slightly greater than the standard deviations of the sounds being compared (Table II).

It is interesting to compare the jnd to the control of a performer in repeating notes with the same frequency. The average of standard deviations of notes in Table III with respect to the same note (unmodulated) is 4.2 ± 3.9 cents. Thus we can speculate that intonation control may be due in part to limits of pitch perception as well as motor control. There is also an inherent uncertainty of about 2 cents in the frequency produced by a bowed instrument due to the bowing mechanism (inhomogeneity of the bow hair, etc.) (McIntyre and Woodhouse, 1978; McIntyre *et al.*, 1981).

All of the data which were taken support the hypothesis that the mean frequency of a modulated sound is perceived when a subject is asked to compare it to an unmodulated sound. Further this modulated sound gives rise to the same psychometric curve as that of a single frequency sound at its mean, i.e., for purposes of comparisons with a second sound it is equivalent to a single frequency sound having its mean frequency. Equivalently we can say that a human functioning as a frequency meter performs identically on an unmodulated sound and the mean of a frequency modulated sound.

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